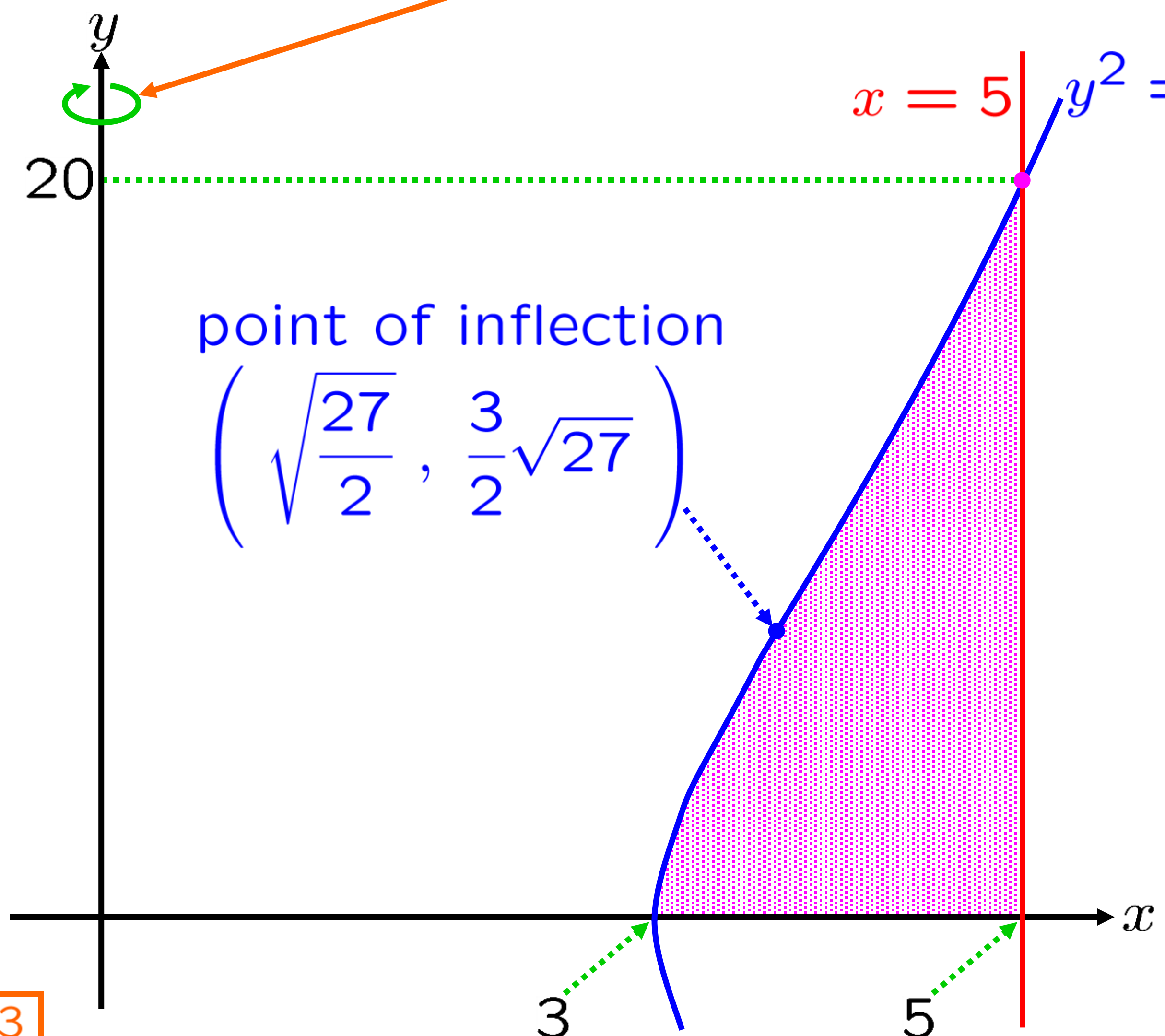
CALCULUS
Volume by cylindrical shells, more problems

EXAMPLE: Using the shell method, find the volume of the solid obtained by revolving, about the y-axis, the region bounded by: $y^2 = x^2(x^2 - 9)$, $y = 0$ and $x = 5$.

Sketch the region.



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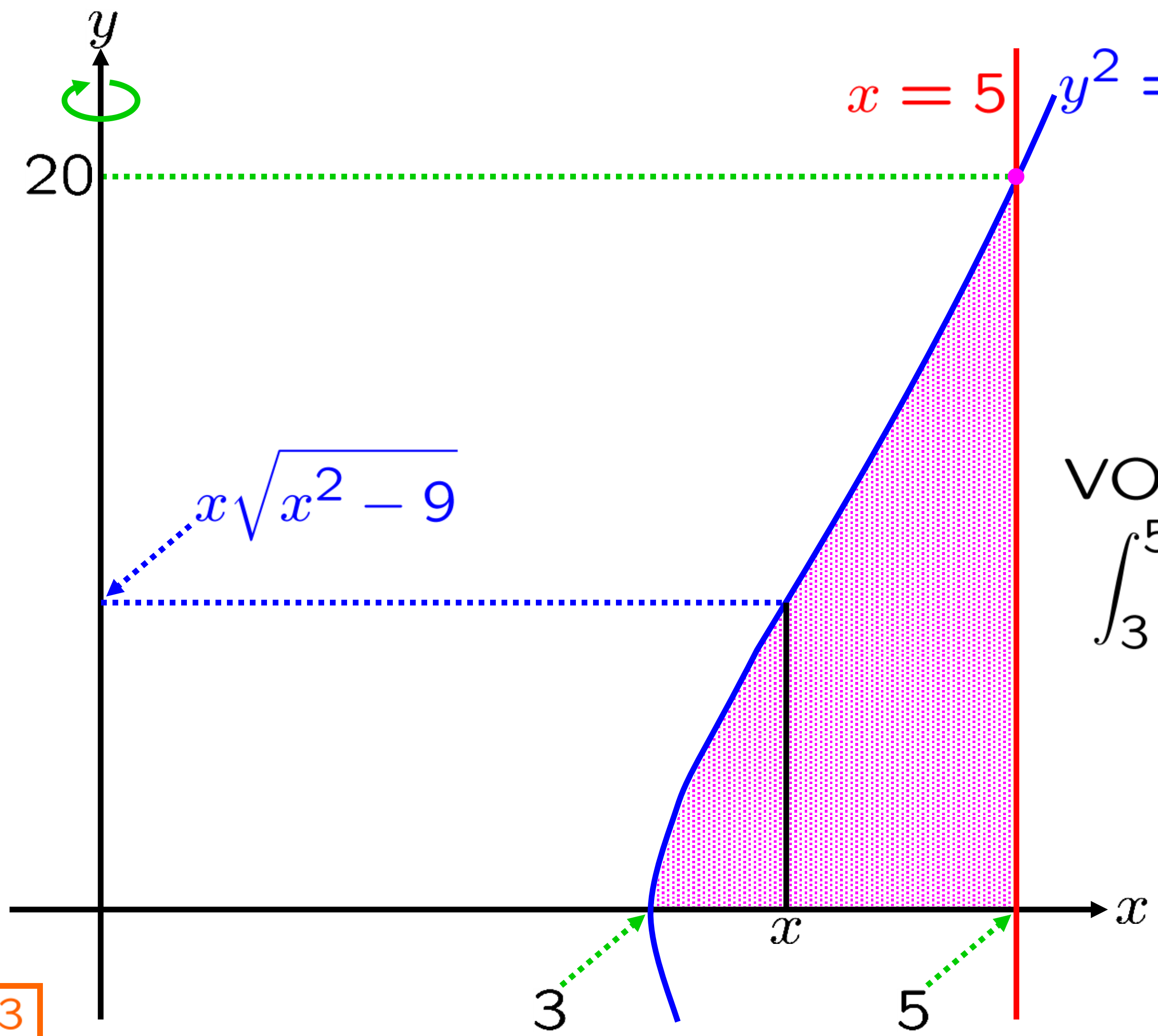

VOLUME =

$$\int_3^5 [2\pi x]\left[x\sqrt{x^2-9}\right]dx$$

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$$\text{VOLUME} = \int_3^5 [2\pi x] \left[x\sqrt{x^2 - 9} \right] dx$$

$$u = x^2 - 9, \; du = 2x \, dx$$

$$\text{VOLUME} = \int_3^5 [2\pi x] \left[x\sqrt{x^2 - 9} \right] dx$$

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Sketch the region.

$$\text{VOLUME} = \int_3^5 [2\pi x]\left[x\sqrt{x^2-9}\right] dx$$

$$= \pi \int_0^{16} \sqrt{u+9}\sqrt{u}\, du$$

$$= \pi \int_{0+\frac{9}{2}}^{16+\frac{9}{2}} \sqrt{v+\frac{9}{2}}\sqrt{v-\frac{9}{2}}\, dv$$

$$= \pi \int_{\frac{2}{9}(0+\frac{9}{2})}^{\frac{2}{9}(16+\frac{9}{2})} \sqrt{\frac{9}{2}w+\frac{9}{2}}\sqrt{\frac{9}{2}w-\frac{9}{2}}\,\frac{9}{2}dw$$

$$= \pi \int_1^{41/9} \sqrt{\frac{9}{2}}\sqrt{w+1}\sqrt{\frac{9}{2}}\sqrt{w-1}\,\frac{9}{2}dw$$

$$= \frac{81\pi}{4}\int_1^{41/9} \sqrt{w^2-1}\, dw$$

$$\sqrt{u+9} = x$$

$$u = x^2 - 9, \quad du = 2x\, dx$$

$$v = u + \frac{9}{2}, \quad dv = du$$

$$v - \frac{9}{2} = u$$

$$w = \frac{2}{9}v, \quad dw = \frac{2}{9}dv$$

$$\frac{9}{2}w = v, \quad \frac{9}{2}dw = dv$$

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Sketch the region.

$$\text{VOLUME} = \frac{81\pi}{4} \int_1^{41/9} \sqrt{w^2 - 1}\, dw$$

$$w = \cos t??$$

$$= \frac{81\pi}{4} \int_1^{41/9} \sqrt{w^2 - 1}\, dw$$

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Sketch the region.

$$\text{VOLUME} = \frac{81\pi}{4} \int_1^{41/9} \sqrt{w^2 - 1}\, dw$$

$$\cos^2 - 1 = -\sin^2$$

$$w = \cos t??$$

$$w = \cosh t$$

$$dw = \sinh t\, dt$$

$$w^2 - 1 = \sinh^2 t$$

$$\sqrt{w^2 - 1} = |\sinh t|$$

provided $t \geq 0$

$$\cosh t := \frac{e^t + e^{-t}}{2}$$

$$\sinh t := \frac{e^t - e^{-t}}{2}$$

$$\cosh^2 - 1 = \sinh^2$$

solve . . .

$$\cosh a = \frac{41}{9},\ a \geq 0$$

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Sketch the region.

$$\text{VOLUME} = \frac{81\pi}{4}\int_1^{41/9}\sqrt{w^2-1}\,dw$$

$$= \frac{81\pi}{4}\int_0^a \sinh^2 t\,dt$$

$$w = \cosh t$$

$$dw = \sinh t\,dt$$

$$w^2 - 1 = \sinh^2 t$$

$$\sqrt{w^2-1} = \sinh t,$$

provided $t \geq 0$

$$\cosh t := \frac{e^t + e^{-t}}{2}$$

$$\sinh t := \frac{e^t - e^{-t}}{2}$$

$$\cosh^2 - 1 = \sinh^2$$

$$b := e^a$$

solve . . . $\cosh a = \frac{41}{9}$, $a \geq 0$

$$\left[\frac{1}{2}\left(b + \frac{1}{b}\right) = \frac{e^a + e^{-a}}{2} = \frac{41}{9}\right] \times b$$

$$\left(\frac{1}{2}\right)b^2 + \frac{1}{2} = \left(\frac{41}{9}\right)b$$

EXAMPLE: Using the shell method, find the volume of the solid obtained by revolving, about the y-axis, the region bounded by: $y^2 = x^2(x^2 - 9)$, $y = 0$ and $x = 5$. Sketch the region.

$$\text{VOLUME} = \frac{81\pi}{4}\int_1^{41/9}\sqrt{w^2-1}\,dw = \frac{81\pi}{4}\int_0^a \sinh^2 t\,dt$$

$$w = \cosh t$$
$$dw = \sinh t\,dt$$
$$w^2 - 1 = \sinh^2 t$$
$$\sqrt{w^2-1} = \sinh t,$$
provided $t \geq 0$

$$\cosh t := \frac{e^t + e^{-t}}{2}$$
$$\sinh t := \frac{e^t - e^{-t}}{2}$$
$$\cosh^2 - 1 = \sinh^2$$

$$b := e^a$$

solve . . . $\cosh a = \frac{41}{9}$, $a \geq 0$

$$\left(\frac{1}{2}\right)b^2 - \left(\frac{41}{9}\right)b + \frac{1}{2} = 0$$

$$\left(\frac{1}{2}\right)b^2 + \frac{1}{2} = \left(\frac{41}{9}\right)b$$

EXAMPLE: Using the shell method, find the volume of the solid obtained by revolving, about the y-axis, the region bounded by: $y^2 = x^2(x^2 - 9)$, $y = 0$ and $x = 5$.

Sketch the region.

$$\text{VOLUME} = \frac{81\pi}{4} \int_1^{41/9} \sqrt{w^2 - 1}\, dw$$

$$= \frac{81\pi}{4} \int_0^{a} \sinh^2 t\, dt$$

$$w = \cosh t$$

$$dw = \sinh t\, dt$$

$$w^2 - 1 = \sinh^2 t$$

$$\sqrt{w^2 - 1} = \sinh t\,,$$

provided $t \geq 0$

$$\cosh t := \frac{e^t + e^{-t}}{2}$$

$$\sinh t := \frac{e^t - e^{-t}}{2}$$

$$\cosh^2 - 1 = \sinh^2$$

$$b := e^a$$

solve . . . $\cosh a = \dfrac{41}{9}$, $a \geq 0$

$e^a \geq e^0$

$$\left(\frac{1}{2}\right) b^2 - \left(\frac{41}{9}\right) b + \frac{1}{2} = 0, \; b \geq 1$$

$$b = \frac{41}{9} \pm \sqrt{\left(\frac{41}{9}\right)^2 - 1} = \cdots$$

EXAMPLE: Using the shell method, find the volume of the solid obtained by revolving, about the y-axis, the region bounded by: $y^2 = x^2(x^2 - 9)$, $y = 0$ and $x = 5$. Sketch the region.

$$\text{VOLUME} = \frac{81\pi}{4} \int_1^{41/9} \sqrt{w^2 - 1}\, dw$$

$$= \frac{81\pi}{4} \int_0^a \sinh^2 t\, dt$$

$$= \frac{81\pi}{4} \int_0^a \left(\frac{e^t - e^{-t}}{2} \right)^2 dt$$

$$w = \cosh t$$

$$dw = \sinh t\, dt$$

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$$\sqrt{w^2 - 1} = \sinh t\,,$$

provided $t \geq 0$

$$\cosh t := \frac{e^t + e^{-t}}{2}$$

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$$\cosh a = \frac{41}{9},\ a \geq 0$$

$$b := e^a$$

$$a = \ln b = \cdots$$

$$b = \frac{41}{9} + \sqrt{\left(\frac{41}{9}\right)^2 - 1} = \cdots$$

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Sketch the region.

$$\text{VOLUME} = \frac{81\pi}{4}\int_0^a \left(\frac{e^t - e^{-t}}{2}\right)^2 dt$$

$$= \frac{81\pi}{4}\int_0^a \frac{(e^t)^2 - 2(e^t)(e^{-t}) + (e^{-t})^2}{4}\, dt$$

$$= \frac{81\pi}{4}\int_0^a \left(\frac{e^t - e^{-t}}{2}\right) dt$$

$$a = \ln b = \cdots$$

$$b = \frac{41}{9} + \sqrt{\left(\frac{41}{9}\right)^2 - 1} = \ln b = \cdots$$

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$$= \frac{81\pi}{\boxed{4}}\int_0^a \frac{(e^t)^2 - 2(e^t)(e^{-t}) + (e^{-t})^2}{\boxed{4}}\,dt$$

$$= \frac{81\pi}{\boxed{16}}\int_0^a e^{2t} - 2 + e^{-2t}\,dt$$

$$= \frac{81\pi}{16}\left[\frac{e^{2t}}{2} - 2t + \frac{e^{-2t}}{-2}\right]_{t:\to 0}^{t:\to a} = \cdots \blacksquare$$

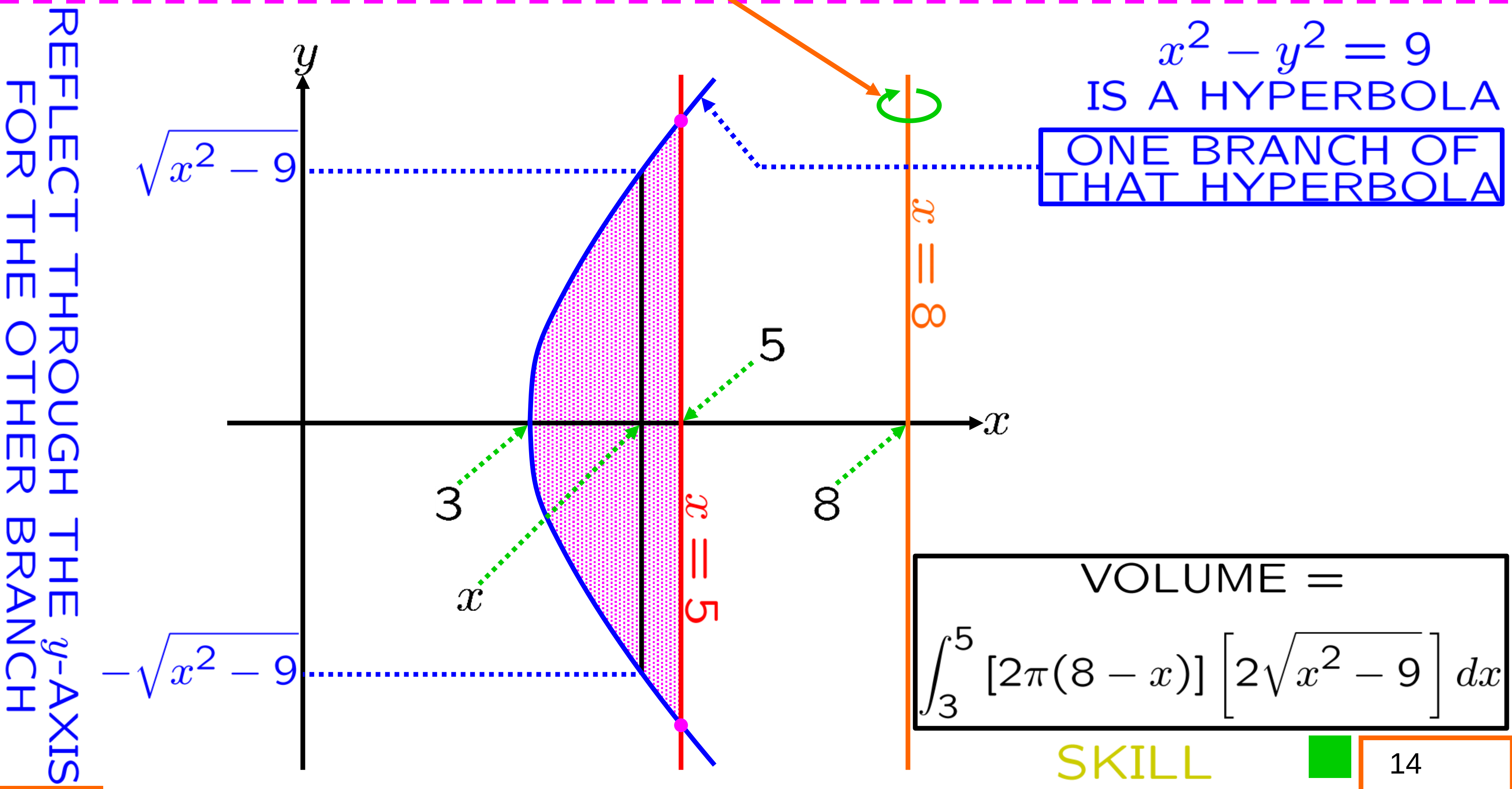
SKILL
shell method

$$b = \frac{41}{9} + \sqrt{\left(\frac{41}{9}\right)^2 - 1} = \cdots \qquad a = \ln b = \cdots$$

EXAMPLE: Using the shell method, set up an integral to compute the volume of the solid obtained by revolving, about the line $x = 8$, the region bounded by:

$$x^2 - y^2 = 9 \quad \text{and} \quad x = 5.$$

Sketch the region. DO NOT work out the integral.

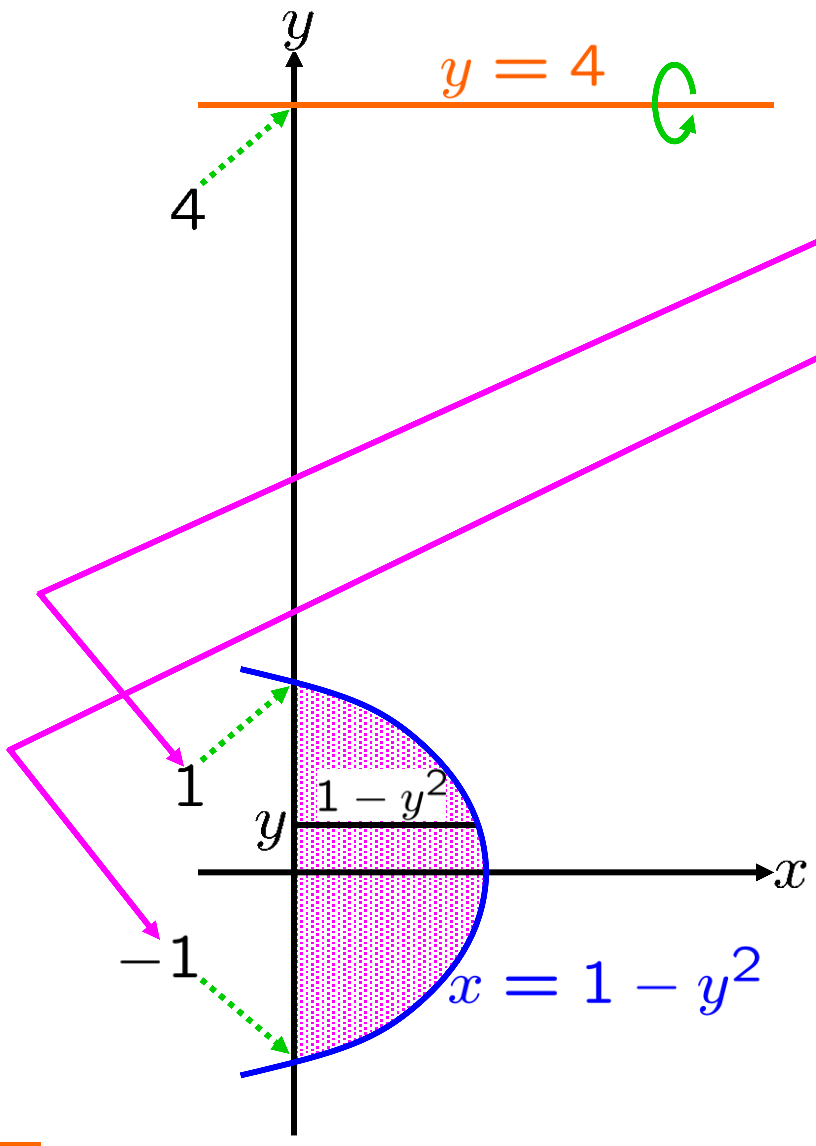


VOLUME =

$$\int_3^5 [2\pi(8 - x)] \left[2\sqrt{x^2 - 9}\right] dx$$

SKILL
shell method

EXAMPLE: Describe the solid whose volume is represented by the integral $\int_{-1}^{1} 2\pi(4-y)(1-y^2)\,dy$.



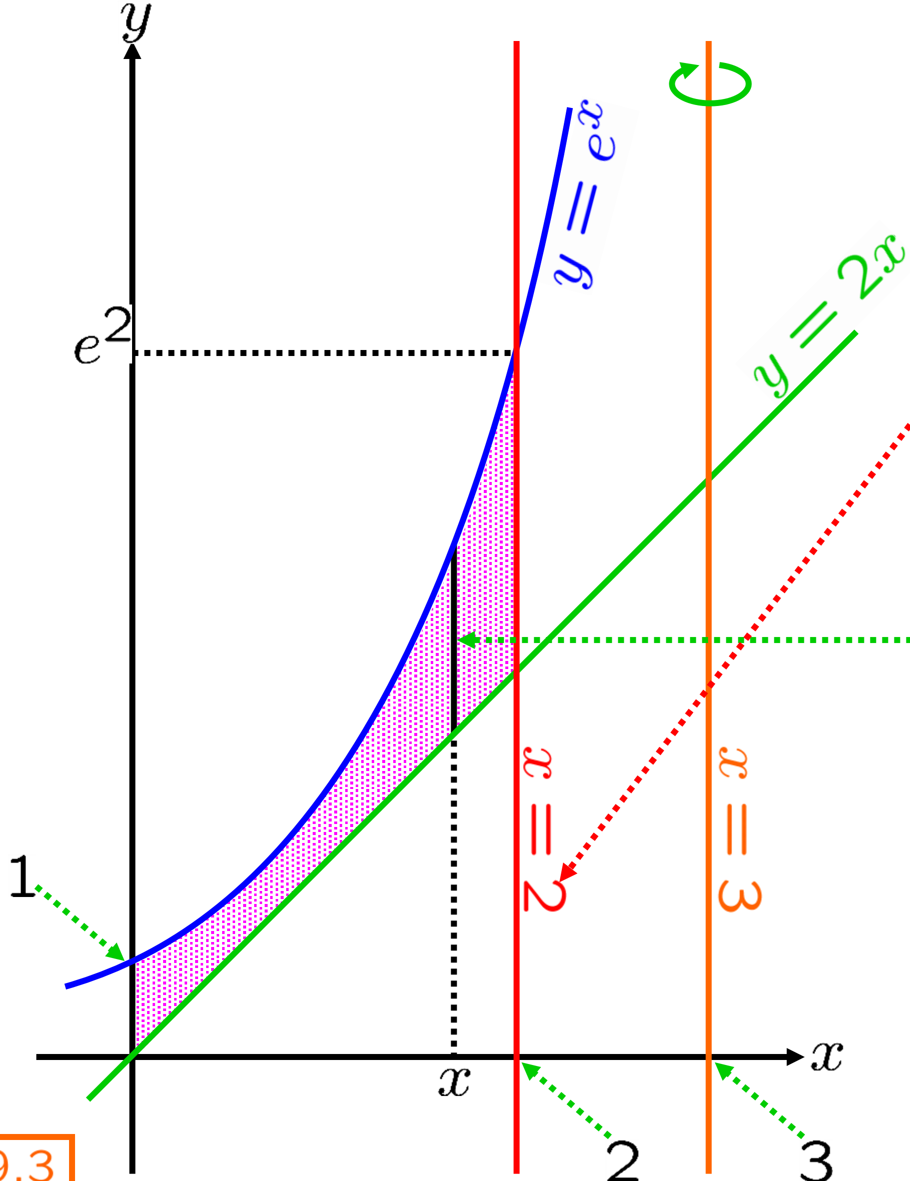
circumference, radius, width

$$\int_{-1}^{1} 2\pi(4-y)(1-y^2)\,dy$$

is the volume of the solid obtained by revolving, about the line $y = 4$, the region bounded by $x = 1 - y^2$ and the y-axis. ■

SKILL
region from integral

EXAMPLE: Describe the solid whose volume is represented by the integral $\int_0^2 2\pi(3-x)(e^x-2x)\,dx$.



circumference

radius

height

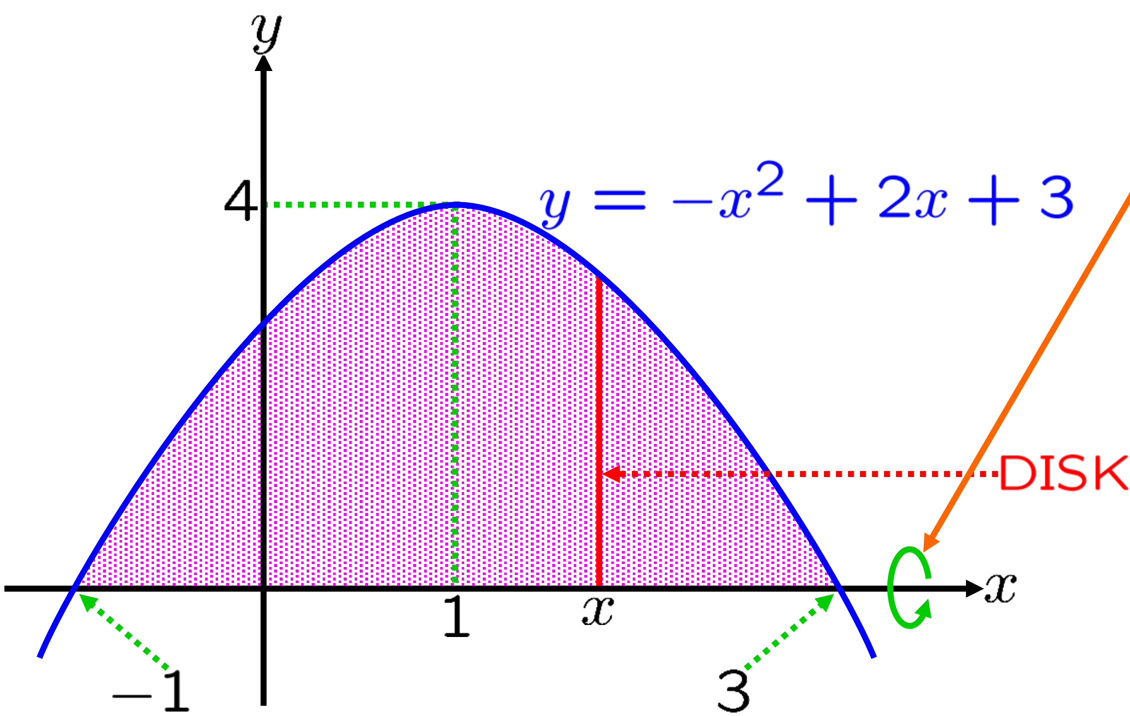
$$\int_0^2 2\pi(3-x)(e^x-2x)\,dx$$

is the volume of the solid obtained by revolving, about the line $x=3$, the region bounded by $y=e^x$, $y=2x$, $x=2$ and the x-axis. ■

SKILL
region from integral

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = \int_{-1}^{3} \pi[-x^2 + 2x + 3]^2 \, dx$$



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$$\text{VOLUME} = \int_{-1}^{3} \pi[-x^2 + 2x + 3]^2 \, dx$$

$$= \pi \int_{-1}^{3} [-x^2 + 2x + 3]^2 \, dx$$

$$= \pi \int_{-1}^{3} (-x^2)^2 + (2x)^2 + 3^2 + 2(-x^2)(2x) + 2(-x^2)(3) + 2(2x)(3) \, dx$$

$$= \pi \int_{-1}^{3} x^4 + 4x^2 + 9 - 4x^3 - 6x^2 + 12x \, dx$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = \pi \int_{-1}^{3} x^4 + 4x^2 + 9 - 4x^3 - 6x^2 + 12x \, dx$$

$$= \pi \int_{-1}^{3} x^4 - 4x^3 - 2x^2 + 12x + 9 \, dx$$

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$$= \pi \int_{-1}^{3} x^4 - 4x^3 - 2x^2 + 12x + 9 \, dx$$

$$= \pi \left[\frac{x^5}{5} - x^4 - \frac{2x^3}{3} + 6x^2 + 9x \right]_{x:\to -1}^{x:\to 3}$$

$$= \pi \left[\frac{243 - (-1)}{5} - (81 - 1) - \frac{2 \cdot (27 - (-1))}{3} + 6 \cdot (9 - 1) + 9 \cdot (3 - (-1)) \right]$$

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$$= \pi\left[\frac{244}{5} - 80 - \frac{56}{3} + 48 + 36\right]$$

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$$= \pi\left[\frac{244}{5} - 80 - \frac{56}{3} + 48 + 36\right]$$

$$= \pi\left[\frac{244}{5} - \frac{56}{3} + 4\right]$$

Next: Same problem, shell method . . .

$$= \pi\left[\frac{244 \cdot 3}{15} - \frac{56 \cdot 5}{15} + \frac{4 \cdot 15}{15}\right] = \frac{512\pi}{15}$$

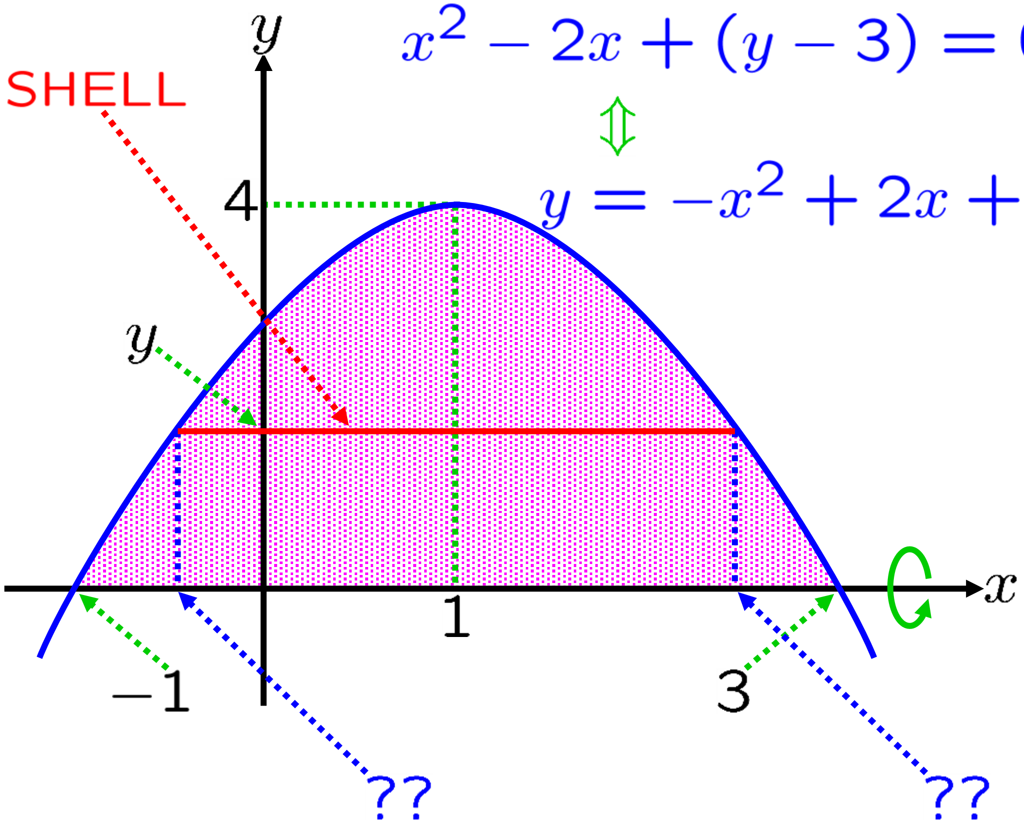
EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$x^2 - 2x + (y-3) = 0 \quad \Leftrightarrow \quad x = \frac{\cancel{2} \pm \sqrt{\cancel{4} - \cancel{4}(y-3)}}{\cancel{2}}$$

$$\Updownarrow$$

$$y = -x^2 + 2x + 3$$

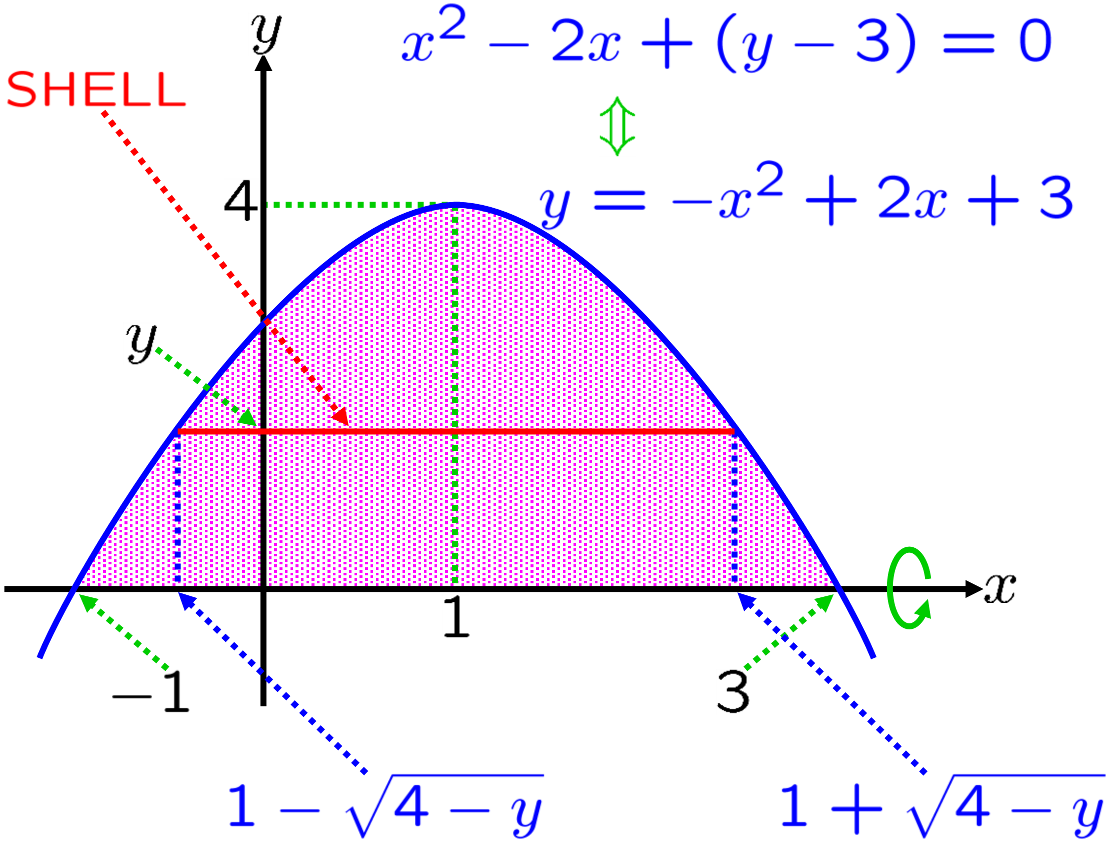
$$x = 1 \pm \sqrt{1 - (y-3)} = 1 \pm \sqrt{4 - y}$$



Next: Same problem, shell method . . .

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = \int_0^4 [2\pi y]\left[\left(1 + \sqrt{4 - y}\right) - \left(1 - \sqrt{4 - y}\right)\right] dy$$



$$x^2 - 2x + (y - 3) = 0 \iff x = \frac{2 \pm \sqrt{4 - 4(y - 3)}}{2}$$

$$\Updownarrow$$

$$y = -x^2 + 2x + 3$$

$$x = 1 \pm \sqrt{1 - (y - 3)} = 1 \pm \sqrt{4 - y}$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = \int_0^4 [2\pi y]\left[\left(1 + \sqrt{4-y}\right) - \left(1 - \sqrt{4-y}\right)\right] dy$$

$$= \int_0^4 [2\pi y]\left[\cancel{1} + \sqrt{4-y} - \cancel{1} + \sqrt{4-y}\right] dy$$

$$\sqrt{\bullet} = (\bullet)^{1/2}$$

$$= \int_0^4 [2\pi y]\left[2\sqrt{4-y}\right] dy$$

$$= 4\pi \int_0^4 y(4-y)^{1/2}\, dy$$

$$u = 4 - y$$
$$y = 4 - u$$
$$dy = (-1)\, du$$

$$= 4\pi \int_4^0 (4-u)u^{1/2}(-1)du$$

§9.3

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = 4\pi \int_4^0 (4-u)u^{1/2}(-1)\,du$$

$$= 4\pi \int_0^4 (4-u)u^{1/2}\,du$$

$$= 4\pi \int_4^0 (4-u)u^{1/2}(-1)\,du$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = 4\pi \int_4^0 (4-u)u^{1/2}(-1)du$$

$$= 4\pi \int_0^4 (4-u)u^{1/2}\,du$$

$$= 4\pi \int_0^4 4u^{1/2} - u^{3/2}\,du$$

$$= 4\pi \left[\frac{4u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2}\right]_{u:\to 0}^{u:\to 4}$$

$$= 4\pi \left[\frac{4 \cdot 4^{3/2}}{3/2} - \frac{4^{5/2}}{5/2}\right]$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = 4\pi \left[\frac{4 \cdot 4^{3/2}}{3/2} - \frac{4^{5/2}}{5/2} \right]$$

$$= 4\pi \left[\frac{4^{5/2}}{3/2} - \frac{4^{5/2}}{5/2} \right]$$

$$= 4\pi \left[\frac{4 \cdot 4^{3/2}}{3/2} - \frac{4^{5/2}}{5/2} \right]$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $y = -x^2 + 2x + 3$ and $y = 0$.

$$\text{VOLUME} = 4\pi \left[\frac{4 \cdot 4^{3/2}}{3/2} - \frac{4^{5/2}}{5/2} \right]$$

$$= 4\pi \left[\frac{4^{5/2}}{3/2} - \frac{4^{5/2}}{5/2} \right]$$

$$= 4\pi \left[\frac{1}{3/2} - \frac{1}{5/2} \right] \left[4^{5/2} \right]$$

$$= 4\pi \left[\frac{2}{3} - \frac{2}{5} \right] \left[4^{5/2} \right]$$

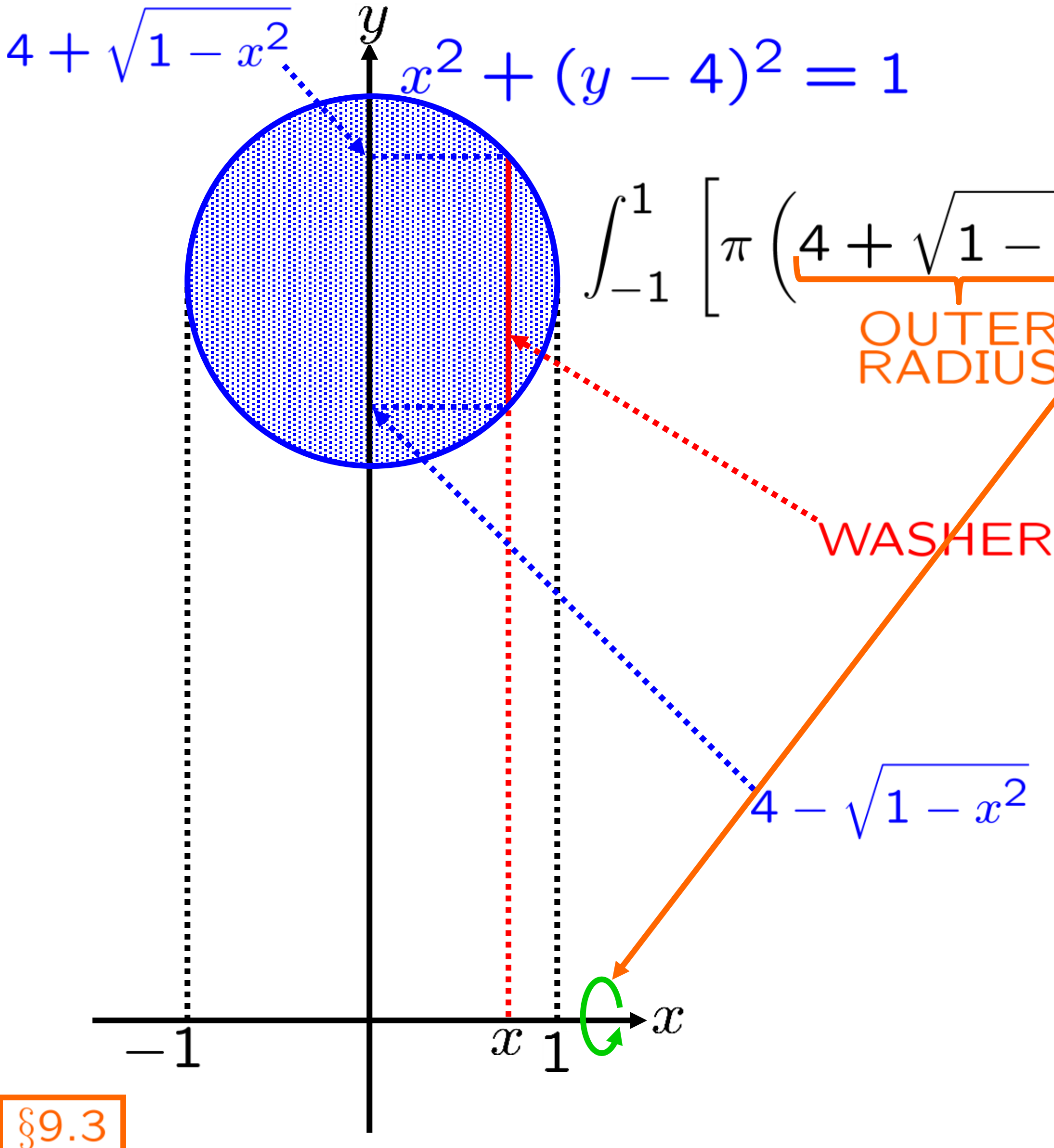
$$= 4\pi \left[\frac{4}{15} \right] \left[(4^{1/2})^5 \right]$$

SKILL
volume of solid

$$= \left[\frac{16\pi}{15} \right] \left[2^5 \right] = \frac{16\pi \cdot 32}{15} = \frac{512\pi}{15}$$

§9.3

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $x^2 + (y-4)^2 = 1$.



VOLUME

||

$$\int_{-1}^{1} \left[\pi \left(\underbrace{4 + \sqrt{1 - x^2}}_{\text{OUTER RADIUS}}\right)^2\right] - \left[\pi \left(\underbrace{4 - \sqrt{1 - x^2}}_{\text{INNER RADIUS}}\right)^2\right] dx$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $x^2 + (y-4)^2 = 1$.

$$\text{VOLUME} = \int_{-1}^{1} \left[\pi\left(4+\sqrt{1-x^2}\right)^2\right] - \left[\pi\left(4-\sqrt{1-x^2}\right)^2\right] dx$$

$$= \pi \int_{-1}^{1} \left(4+\sqrt{1-x^2}\right)^2 - \left(4-\sqrt{1-x^2}\right)^2 dx$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $x^2 + (y-4)^2 = 1$.

$$\text{VOLUME} = \int_{-1}^{1} \left[\pi\left(4+\sqrt{1-x^2}\right)^2\right] - \left[\pi\left(4-\sqrt{1-x^2}\right)^2\right] dx$$

$$= \pi \int_{-1}^{1} \left(4+\sqrt{1-x^2}\right)^2 - \left(4-\sqrt{1-x^2}\right)^2 dx$$

$$= 16\pi \int_{-1}^{1} \sqrt{1-x^2}\, dx$$

SUBTRACT

$$\left(4+\sqrt{1-x^2}\right)^2 = 16 + 8\sqrt{1-x^2} + (1-x^2)$$

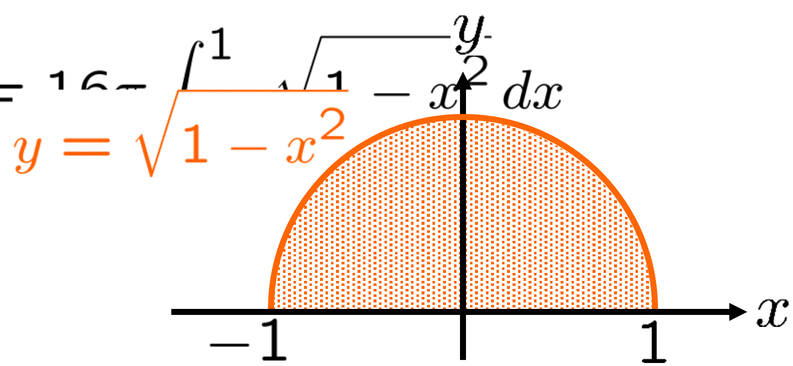
$$\left(4-\sqrt{1-x^2}\right)^2 = 16 - 8\sqrt{1-x^2} + (1-x^2)$$

$$\left(4+\sqrt{1-x^2}\right)^2 - \left(4-\sqrt{1-x^2}\right)^2 = 16\sqrt{1-x^2}$$

EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $x^2 + (y-4)^2 = 1$.

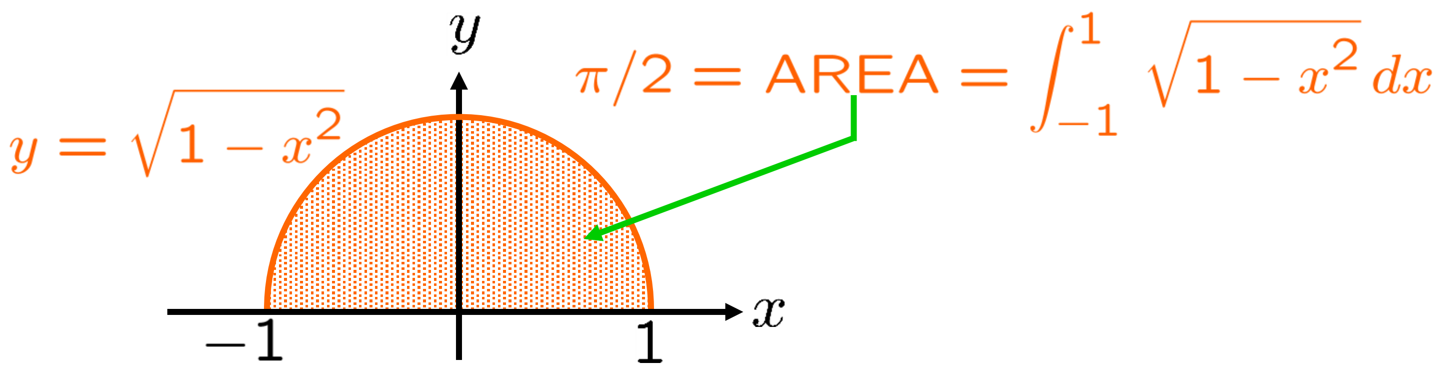
$$\text{VOLUME} = 16\pi \int_{-1}^{1} \sqrt{1-x^2}\,dx$$

$$= 16\pi \int_{-1}^{1} \sqrt{1-x^2}\,dx$$



EXAMPLE: Using *WHATEVER* method you prefer, find the volume generated by revolving, about the x-axis, the region bounded by: $x^2 + (y-4)^2 = 1$.

$$\text{VOLUME} = 16\pi \int_{-1}^{1} \sqrt{1-x^2}\,dx = 16\pi(\pi/2) = 8\pi^2$$



Next: Same problem, shell method . . .

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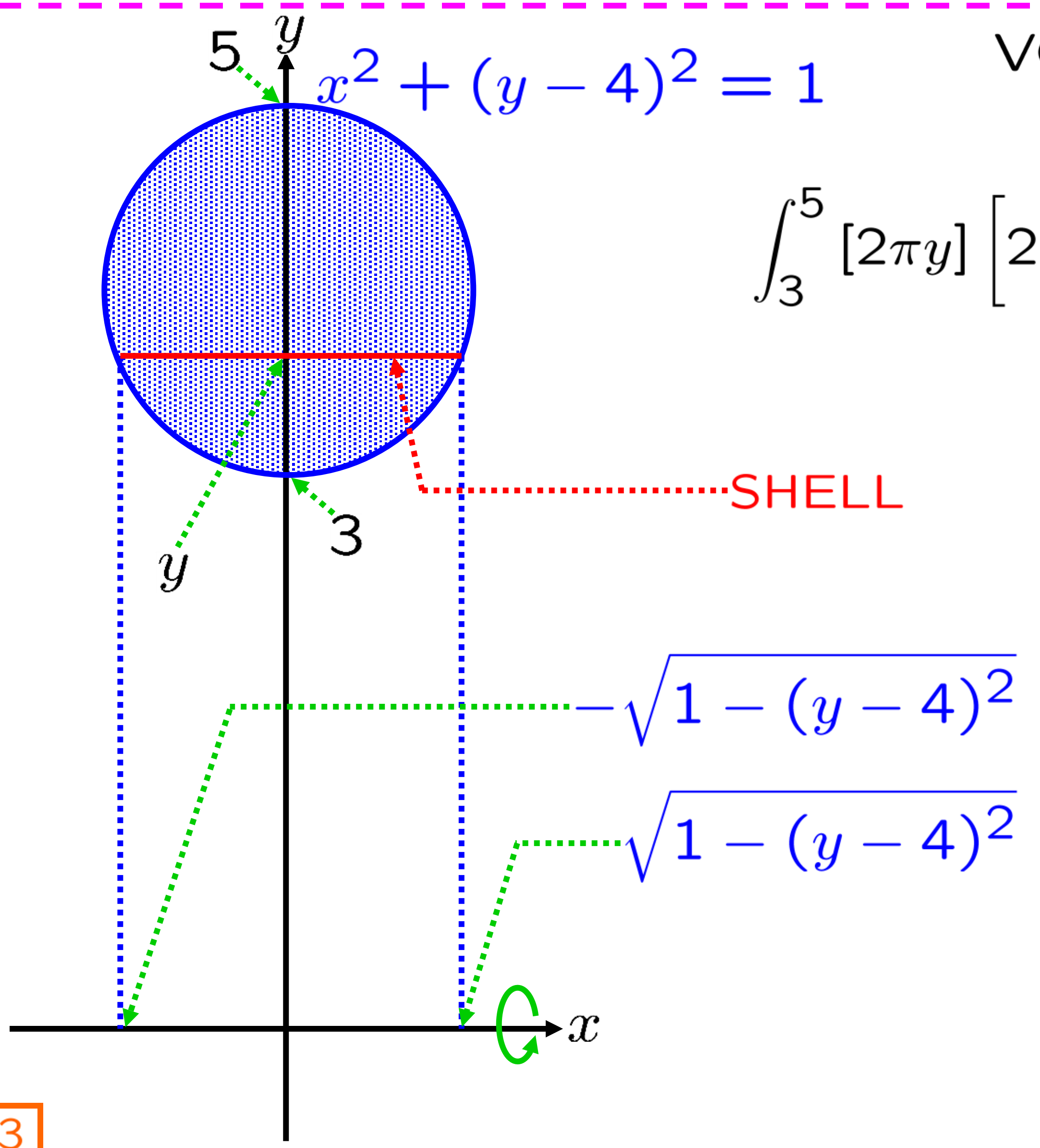

VOLUME

||

$$\int_3^5 [2\pi y] \left[2\sqrt{1-(y-4)^2}\right] dy$$

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VOLUME

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$$u + 4 = y$$
$$u = y - 4$$
$$du = dy$$

$$= 4\pi \int_{-1}^1 (u+4)\sqrt{1-u^2}\, du$$

$$= 4\pi \int_{-1}^1 \left[u\sqrt{1-u^2}\right] + \left[4\sqrt{1-u^2}\right] du$$

$$= 4\pi \left(\left[\int_{-1}^1 u\sqrt{1-u^2}\, du\right] + 4\left[\int_{-1}^1 \sqrt{1-u^2}\, du\right]\right)$$

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$$= 4\pi\left(\left[\int_{-1}^{1} u\sqrt{1-u^2}\,du\right] + 4\left[\int_{-1}^{1} \sqrt{1-u^2}\,du\right]\right)$$

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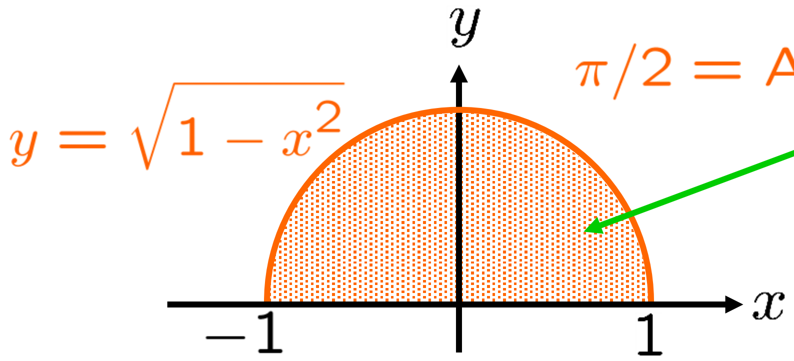
$$= 4\pi\,(0 + 4\,[\pi/2]) = 8\pi^2 \blacksquare$$

SKILL
volume of solid



$$\pi/2 = \text{AREA} = \int_{-1}^{1} \sqrt{1-x^2}\,dx$$

$x :\to u$

$$\pi/2 = \int_{-1}^{1} \sqrt{1-u^2}\,du$$

SKILL

volume of solid

Whitman problems

§9.3, p. 191-192, #1-13

STOP