Central Limit Theorem and Finance
St Catherine's
2 December 2008

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Applied Coin-Flipping

$N = 10^{10^{100}}$ $\quad$ N coin flips $\quad$ H heads, T tails

Male height (inches): $69 + 5\frac{H-T}{\sqrt{N}}$

Probability that: $69 - 5 \leq \text{ht} \leq 69 + 5$?

$$\not{69} - 5 \leq \not{69} + 5\frac{H-T}{\sqrt{N}} \leq \not{69} + 5$$

$$-5 \leq 5\frac{H-T}{\sqrt{N}} \leq 5$$

DIVIDE BY 5

$$-1 \leq \frac{H-T}{\sqrt{N}} \leq 1$$

Applied Coin-Flipping

$N = 10^{10^{100}}$ N coin flips H heads
T tails

Male height (inches): $69 + 5\frac{H-T}{\sqrt{N}}$

Probability that: $69 - 5 \leq \text{ht} \leq 69 + 5$?

Probability that: $-1 \leq \frac{H-T}{\sqrt{N}} \leq 1$?

$$-1 \leq \frac{H-T}{\sqrt{N}} \leq 1$$

Applied Coin-Flipping

$N = 10^{10^{100}}$ N coin flips H heads
T tails

Male height (inches): $69 + 5\dfrac{H - T}{\sqrt{N}}$ square root

Probability that: $69 - 5 \le \text{ht} \le 69 + 5$?

Probability that: $-1 \le \dfrac{H - T}{\sqrt{N}} \le 1$? Answer: $\approx 68\%$

Grav accel (ft/sec^2): $32 + 10^6\dfrac{H - T}{N}$ NO square root

Probability that: $32 - \dfrac{10^6}{\sqrt{N}} \le \text{acc} \le 32 - \dfrac{10^6}{\sqrt{N}}$?

EXTREMELY small

Probability that: $-1 \le \dfrac{H - T}{\sqrt{N}} \le 1$? Answer: $\approx 68\%$

Applied Coin-Flipping

$N =$ number of seconds in 30 days

Current stock price: 1 USD

$$x_+ := \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

$S :=$ stock price 30 days from now

Contract pays: $(S-1)_+$ USD, 30 days from now

Expected payout?

Each second, price changes either by a factor of 1.000035616 or by a factor of 0.999964386.

50% chance of uptick,
50% chance of downtick.

Applied Coin-Flipping

Coin-flipping game: Flip a fair coin N times.
If H heads and T tails,
pay $(u^H d^T - 1)_+$,
30 days from now.

Contract pays: $(S-1)_+$ USD,
30 days from now

Expected payout?

Each second, price changes
either by a factor of 1.000035616 $=: u$
or by a factor of 0.999964386. $=: d$

50% chance of uptick,
50% chance of downtick.

Applied Coin-Flipping

Coin-flipping game: Flip a fair coin N times.
If H heads and T tails,
pay $(u^H d^T - 1)_+$,
30 days from now.

Expected payout?

Computing probabilities is relatively easy,

Expected payout?

computing expected values is generally harder.

Compute the probability that

$$-1 < \frac{H-T}{\sqrt{N}} < 1.$$

$$X := (H-T)/\sqrt{N}$$

Compute the probability that

$$-1 < X < 1.$$

$$\begin{aligned} H_1 &:= \text{number of heads after first flip} \\ H_2 &:= \text{number of heads after second flip} \\ &\vdots \\ H_N &:= \text{number of heads after } N\text{th flip} = H \end{aligned}$$

Compute the probability that

$$-1 < \frac{H-T}{\sqrt{N}} < 1.$$

$$X := (H-T)/\sqrt{N}$$

Compute the probability that

$$-1 < X < 1.$$

X is *hard* . . .

For all integers $j \in [1, N]$,

$H_j :=$ number of heads after jth flip

$T_j :=$ number of tails after jth flip

$D_j := H_j - T_j$

Easier: D_1, $D_1/7$, D_2, D_N

$H = H_N$, $T = T_N$,

$$X = (H_N - T_N)/\sqrt{N} = D_N/\sqrt{N}$$

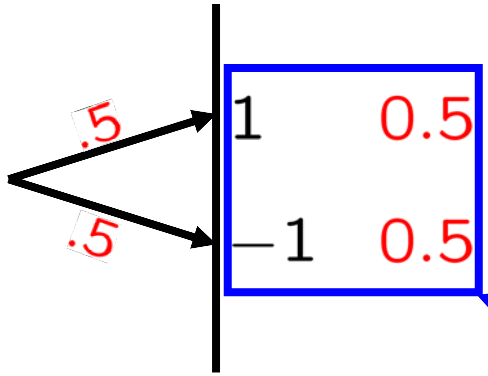
$D_1 = \overset{0}{H_1} - \overset{1}{T_1}$:

random variable

a variable whose value is determined by random events

(probability) measure of D_1

(probability) distribution of D_1



distribution of $T_1 - H_1$ is exactly the same

keep the distribution

forget its origin

divide by 7

D_1 ~~$= H_1 - T_1$~~ :

What about $D_1/7$?

Cannot recover the ra[ndom variable.]
Only its distribution.

0.5	1
0.5	−1

$z^1 = z$

z^{-1}

Fourier transform of the distribution of D_1 is $\cos t$

$i = \sqrt{-1}$

$T_1 - H_1$ has the same distribution.

Replace z by e^{-it}

Generating ~~function~~ (expression of z):

~~$(0.5)z + (0.5)z^{-1}$~~

Fourier transform:

~~$(0.5)e^{-it} + (0.5)e^{it}$~~

ξ t not time

$\|$

$\cos t$

keep the distribution
forget its origin

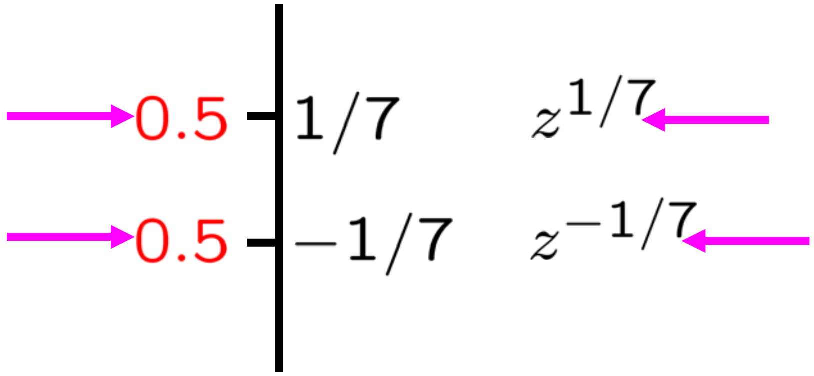
Repl. t by $t/7$

$0.5 \times [\; e^{it} = \cos t + i \sin t \;] +$

$0.5 \times [\; e^{-it} = \cos t - i \sin t \;]$

Inverse Fourier transform

$D_1/7$:



What about $D_1/7$?

Replace t by $t/7$.

$i = \sqrt{-1}$

Replace z by e^{-it}

Generating function: $(0.5)z^{1/7} + (0.5)z^{-1/7}$

Fourier transform: $(0.5)e^{-it/7} + (0.5)e^{it/7}$

$$\| $$

$$\cos(t/7)$$

$$e^{it/7} = \cos(t/7) + i\,\sin(t/7)$$
$$e^{-it/7} = \cos(t/7) - i\,\sin(t/7)$$

$D_2 = \overset{0}{H_2} - \overset{2}{T_2}$:

forget its origin keep the distribution

$D_2 = H_2 - T_2$:

0.25 — 2 — z^2

0.5 — 0 — $z^0 = 1$

0.25 — -2 — z^{-2}

forget its origin keep the distribution

Generating function:

$$(0.25)z^2 + 0.5 + (0.25)z^{-2}$$
$$= ((0.5)z + (0.5)z^{-1})^2$$

the generating function of the distribution of D_1

$$i = \sqrt{-1}$$

Replace z by e^{-it}

Fourier transform: $(\cos t)^2 = \cos^2 t$

$D_N = \sout{H_N - T_N}$:

divide by $\sqrt{N}$

NO WAY!!

Goal: $X =$

What about $D_N/\sqrt{N}$?

Replace t by $t/\sqrt{N}$.

Generating function:

NO WAY!!

$$= ((0.5)z + (0.5)z^{-1})^N$$

the generating function
of the distribution
of D_1

$i = \sqrt{-1}$

Replace z by e^{-it}

Fourier transform: $(\cos t)^N = \cos^N t$

$X = D_N/\sqrt{N}$:

Goal: X =

What about $D_N/\sqrt{N}$?

Replace t by $t/\sqrt{N}$.

NO WAY!!

Fourier transform: $\cos^N(t/\sqrt{N})$

$X = D_N/\sqrt{N}$:

Generating functions
Fourier transforms

NO WAY!!

Fourier transform: $\cos^N(t/\sqrt{N})$

Fourier transform: $\cos^N(t/\sqrt{N})$

$X = D_N/\sqrt{N}$:

Generating functions
Fourier transforms
Fourier analysis
Spectral theory

Useful?

NO WAY!!

The problem:

Compute the probability that

$$-1 < X < 1.$$

Exercise: $\lim_{n\to\infty} \cos^n(3/\sqrt{n}) = e^{-3^2/2}$

Fourier transform: $\cos^N(t/\sqrt{N})$

$$\approx \lim_{n\to\infty} \cos^n(t/\sqrt{n}) = e^{-t^2/2}$$

Verify for $t = 3$.

$X = D_N/\sqrt{N}$:

Fourier transform: $\quad \cos^N(t/\sqrt{N})$

$$\approx \lim_{n\to\infty} \cos^n(t/\sqrt{n}) = e^{-t^2/2}$$

Fourier transform: $\quad \cos^N(t/\sqrt{N})$

$$\approx \lim_{n\to\infty} \cos^n(t/\sqrt{n}) = e^{-t^2/2}$$

$X = D_N/\sqrt{N}$:

Fourier transform: $\cos^N(t/\sqrt{N})$

$$\approx \lim_{n \to \infty} \cos^n(t/\sqrt{n}) = e^{-t^2/2}$$

Key idea of Central Limit Theorem:

Let Z have distr. with Fourier transf. $e^{-t^2/2}$.

Then Z is "close" to X.

in di How to find Z?

Inverse Fourier Transform

The problem:

Compute the probability that

$$-1 < X < 1.$$

Approximately equal to the probability that

$$-1 < Z < 1.$$

Z:

$\frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$ infinitesimal	x	Do this for all $x \in \mathbb{R}$

∃ RV Z with this dist.

NOTES

Mistake:

$$\int_{-\infty}^{\infty} e^{-x^2/2}\, dx = \sqrt{2\pi}$$

$D_2 \in \{2, 0, -2\}$

distribution supported on three points

$D_N \in \{-N, -N+2, \ldots, N-2, N\}$

distribution supported on $N+1$ points

By contrast, the distribution of Z does not have finite support.

Z:

$$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx \quad x$$

Do this for *all* $x \in \mathbb{R}$

Problem: Compute the probability that

$$Z = 7$$

Solution: $$\int_7^7 \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = 0$$

Z:

$$\frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}\, dx \; \Big|\; x$$

Do this for *all* $x \in \mathbb{R}$

Problem: Compute the probability that

$$2 < Z < 3$$

Solution: $$\int_2^3 \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}\, dx = [\Phi(x)]_{x=2}^{x=3}$$

$$= \Phi(3) - \Phi(2) = 0.0214$$

$$= 2.14\%$$

Z:

$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$ | x $\quad z^x$

Do this for *all* $x \in \mathbb{R}$

Generating function:

$$\int_{-\infty}^{\infty} z^x \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx = \text{Exercise}$$

Fourier transform:

Verify for $t = 3i$.

$$\int_{-\infty}^{\infty} e^{-itx} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx = e^{-t^2/2}$$

Key idea of Central Limit Theorem:

Let Z have distr. with Fourier transf. $e^{-t^2/2}$.

Then Z is "close" to X.

Z:

$\wr\wr$

X

$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$ | x $\quad z^x$

Do this for all $x \in \mathbb{R}$

Exercise: $\int_{-\infty}^{\infty} e^{3x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = e^{3^2/2}$

Fourier transform:

Verify for $t = 3i$.

$$\int_{-\infty}^{\infty} e^{-itx} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = e^{-t^2/2}$$

Key idea of Central Limit Theorem:

Let Z have distr. with Fourier transf. $e^{-t^2/2}$.

Then Z is "close" to X.

Z:

$\approx$

X

$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$ | x z^x

probability problems, *then* expected value problems

Do this for *all* $x \in \mathbb{R}$

The problem:

Compute the probability that
$-1 < X < 1$.

Approximately equal to the probability that
$-1 < Z < 1$.

Approximate solution:

Berry-Esseen Theorem

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx = [\Phi(x)]_{x=-1}^{x=1}$$
$$= 68.27\%$$

Goal:

Compute the expected value of $f(u^H d^T)$.

Coin-flipping game: Flip a fair coin N times.
If H heads and T tails,
pay $(u^H d^T - 1)_+$,
30 days from now.

$$f(x) = (x - 1)_+$$

$$f(x) = (x-1)_+$$

Goal:

Compute the expected value of $f(u^H d^T)$.

New easier problem:

Compute the expected value of $f(D_2)$.

$D_2 = H_2 - T_2$:

0.25	2	$f(2)$
0.5	0	$f(0)$
0.25	-2	$f(-2)$

$f :\to g$

works for any function

$$[0.25][f(2)] + [0.5][f(0)] + [0.25][f(-2)] = 1,250$$

Define: $g(x) = 5e^x + x^2$

Goal:

Compute the expected value of $f(u^H d^T)$.

New easier problem:

Compute the expected value of $g(D_2)$.

$D_2 = H_2 - T_2$:

0.25	2	$g(2)$
0.5	0	$g(0)$
0.25	-2	$g(-2)$

$[0.25][g(2)] \ + \ [0.5][g(0)] \ + \ [0.25][g(-2)] \ =$ Exercise

Recall: $f(x) = (x-1)_+$

Goal:

Compute the expected value of $f(u^H d^T)$.

New easier problem:

Compute the expected value of $f(Z)$.

Z:	
$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$	x $f(x)$

Do this for *all* $x \in \mathbb{R}$

$$\int_{-\infty}^{\infty} [f(x)] \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$$

$f(x) = (x-1)_+$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f(x)]\, e^{-x^2/2}\,dx = \text{exercise}$$

Recall: $f(x) = (x-1)_+$

Goal:

Compute the expected value of $f(u^H d^T)$.

New easier problem:

Compute the expected value of $f(X)$.

Z: $\approx$ X

$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$ — x $f(x)$

Do this for all $x \in \mathbb{R}$

$$\int_{-\infty}^{\infty} [f(x)] \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx$$

$f(x) = (5000x - 5000)_+$

Approx. Sol'n: $= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f(x)]\, e^{-x^2/2}\,dx$ = exercise

Recall: $f(x) = (x-1)_+$

write H, T as expr.s of X

Goal:

Compute the expected value of $\boxed{f(u^H d^T)}$.

$\|$?

New easier problem:

Compute the expected value of $\boxed{g(X)}$.

Approx. Sol'n: $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)]\, e^{-x^2/2}\, dx$

Recall: $f(x) = (x-1)_+$

write H, T as expr.s of X

Goal:

Compute the expected value of $f(u^H d^T)$.

New easier problem:

Compute the expected value of $g(X)$.

$X = (H-T)/\sqrt{N}$ $\quad N = 2,592,000$

$\times\sqrt{N}$ $\quad \times\sqrt{N}$

$H + T = N$ $\quad H + T = N$

$H - T = X\sqrt{N}$ ADD NEGATE $\quad -H + T = -X\sqrt{N}$ ADD

$2H = N + X\sqrt{N}$ $\quad 2T = N - X\sqrt{N}$

Approx. Sol'n: $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)]\, e^{-x^2/2}\, dx$

Recall: $f(x) = (x-1)_+$

write H, T as expr.s of X

Goal:

Compute the expected value of $f(u^H d^T)$.

$\|?$

New easier problem:

Compute the expected value of $g(X)$.

$$H = N/2 + X\sqrt{N}/2 \qquad T = N/2 - X\sqrt{N}/2$$

$$u^H = u^{N/2} u^{X\sqrt{N}/2} \qquad d^T = d^{N/2} d^{-X\sqrt{N}/2}$$

$$u^H d^T =$$

$$N := 30 \times 24 \times 60 \times 60 = 2,592,000$$

$$2H = N + X\sqrt{N} \qquad 2T = N - X\sqrt{N}$$

Approx. Sol'n: $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)]\, e^{-x^2/2}\, dx$

Recall: $f(x) = (x-1)_+$

Goal:

Compute the expected value of $f(u^H d^T)$.

$\| ?$

New easier problem:

Compute the expected value of $g(X)$.

$$H = N/2 + X\sqrt{N}/2 \qquad T = N/2 - X\sqrt{N}/2$$

$$u^H = u^{N/2} u^{X\sqrt{N}/2} \qquad d^T = d^{N/2} d^{-X\sqrt{N}/2}$$

$$u^H d^T = u^{N/2}\, d^{N/2}\ \ u^{X\sqrt{N}/2}\, d^{-X\sqrt{N}/2}$$

$$= (ud)^{N/2}\ \ (u/d)^{X\sqrt{N}/2}$$

$$= C\ \ e^{kX}$$

$$C := (ud)^{N/2}$$

$$k := \ln((u/d)^{\sqrt{N}/2})$$

Approx. Sol'n: $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)]\, e^{-x^2/2}\, dx$

$$e^k = (u/d)^{\sqrt{N}/2}$$

Recall: $f(x) = (x-1)_+$

Goal:

Compute the expected value of $f(u^H d^T)$.

$\|$

New easier problem:

Compute the expected value of $g(X)$.

$$g(x) := f(Ce^{kx})$$

$$f(u^H d^T) = f(Ce^{kX}) = g(X)$$

$$\begin{aligned} u^H d^T &= u^{N/2}\, d^{N/2}\;\; u^{X\sqrt{N}/2}\, d^{-X\sqrt{N}/2} \\ &= (ud)^{N/2}\; (u/d)^{X\sqrt{N}/2} \\ &= C\;\, e^{kX} \end{aligned}$$

$$C := (ud)^{N/2}$$

$$k := \ln((u/d)^{\sqrt{N}/2})$$

Approx. Sol'n: $$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)]\, e^{-x^2/2}\, dx$$

Recall: $f(x) = (x-1)_+$

Goal:

Compute the expected value of $f(u^H d^T)$.

$\|$

Restatement of goal:

Compute the expected value of $g(X)$.

$$g(x) := f(Ce^{kx})$$

$$f(u^H d^T) = f(Ce^{kX}) = g(X)$$

$$\begin{aligned} u^H d^T &= u^{N/2}\, d^{N/2}\; u^{X\sqrt{N}/2}\, d^{-X\sqrt{N}/2} \\ &= (ud)^{N/2}\, (u/d)^{X\sqrt{N}/2} \\ &= C\; e^{kX} \end{aligned}$$

$$C := (ud)^{N/2}$$

$$k := \ln((u/d)^{\sqrt{N}/2})$$

Approx. Sol'n:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)]\, e^{-x^2/2}\, dx$$

Recall: $f(x) = (x - 1)_+$

$g(x) := f(Ce^{kx}) = (Ce^{kx} - 1)_+$

$$\left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1)_+ e^{-x^2/2}\, dx\right]$$

k = 0.0573390439

C = 1.000948567

$N = 2,592,000$

u = 1.00010005

d = 0.99989997

$C := (ud)^{N/2}$

$k := \ln((u/d)^{\sqrt{N}/2})$

Approx. Sol'n: $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)]\, e^{-x^2/2}\, dx$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1)_+ \, e^{-x^2/2} \, dx$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty} (Ce^{kx} - 1)_+ \, e^{-x^2/2} \, dx$$

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(Ce^{kx}-1)\,e^{-x^2/2}\,dx$$

$C = 1.000948567$, $k = 0.0573390439$

$$= \frac{1}{\sqrt{2\pi}}\int_{a}^{\infty}(Ce^{kx}-1)\,e^{-x^2/2}\,dx$$

$$Ce^{ka} - 1 = 0$$

$$Ce^{ka} = 1$$

$$e^{ka} = 1/C$$

$$ka = \ln(1/C) = -\ln C$$

$$a = -(\ln C)/k$$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1)_+ \, e^{-x^2/2} \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} (Ce^{kx} - 1) \, e^{-x^2/2} \, dx$$

$$= \frac{5000}{\sqrt{2\pi}} \left[C \int_{a}^{\infty} e^{kx} \, e^{-x^2/2} \, dx \; - \int_{a}^{\infty} e^{-x^2/2} \, dx \right]$$

$$a = -(\ln C)/k$$

$$= \frac{1}{\sqrt{2\pi}} \left[C \int_a^\infty e^{kx} e^{-x^2/2}\, dx \; - \overbrace{\int_a^\infty e^{-x^2/2}\, dx}^{\sqrt{2\pi}\,\Phi(-a)} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[C \int_a^\infty e^{kx} e^{-x^2/2}\, dx \; - \int_a^\infty e^{-x^2/2}\, dx \right]$$

$$a = -(\ln C)/k$$

DON'T FORGET $\sqrt{2\pi}\,\Phi(-a)$ — NEGATE THE LOWER LIMIT

$$= \frac{1}{\sqrt{2\pi}}\left[C\int_a^\infty e^{kx} e^{-x^2/2}\,dx \;-\; \overbrace{\int_a^\infty e^{-x^2/2}\,dx}^{\sqrt{2\pi}\,\Phi(-a)} \right]$$

THE LOWER LIMIT

$$\int_{a-k}^\infty e^{k(x+k)} e^{-(x+k)^2/2}\,dx$$

$$e^{kx}\, e^{k^2}\, e^{-x^2/2}\, e^{-k^2/2}\, e^{-kx}$$

$$e^{k^2/2}\int_{a-k}^\infty e^{-x^2/2}\,dx$$

THE LOWER LIMIT

$$\sqrt{2\pi}\,\Phi(k-a)$$

DON'T FORGET

NEGATE THE LOWER LIMIT

$$a = -(\ln C)/k$$

$$= \frac{1}{\sqrt{2\pi}}\left[C \underbrace{\int_a^\infty e^{kx} e^{-x^2/2}\, dx}_{\int_{a-k}^{\infty} \underbrace{e^{k(x+k)} e^{-(x+k)^2/2}}_{e^{k^2} e^{-x^2/2} e^{-k^2/2}} dx} - \overbrace{\int_a^\infty e^{-x^2/2}\, dx}^{\sqrt{2\pi}\Phi(-a)} \right]$$

$$\underbrace{\qquad\qquad}$$

$$e^{k^2/2} \underbrace{\qquad\qquad}_{\sqrt{2\pi}\Phi(k-a)}$$

$$a = -(\ln C)/k$$

$$= \frac{1}{\sqrt{2\pi}}\left[C \underbrace{\int_a^\infty e^{kx}\, e^{-x^2/2}\, dx} - \overbrace{\int_a^\infty e^{-x^2/2}\, dx}^{\sqrt{2\pi}\,\Phi(-a)} \right]$$

$$e^{k^2/2}\sqrt{2\pi}\,\Phi(k-a)$$

$$a = -(\ln C)/k$$

$$= \frac{1}{\sqrt{2\pi}}\left[C \underbrace{\int_a^\infty e^{kx} e^{-x^2/2}\, dx}_{e^{k^2/2}\sqrt{2\pi}\,\Phi(k-a)} - \overbrace{\int_a^\infty e^{-x^2/2}\, dx}^{\sqrt{2\pi}\,\Phi(-a)} \right]$$

$$= \left[\overbrace{C\, e^{k^2/2}}^{1.002595363} [\Phi(\overbrace{k-a}^{0.073874328})] - [\Phi(\overbrace{-a}^{0.01653528434})] \right]$$

$$= 0.024214088$$

☺

$a = -0.01653528434$

$k = 0.0573390439$

$C = 1.000948567$

$a = -(\ln C)/k$

SUMMARY:

Coin flipping problems are tractable via CLT,
and useful in many applied settings,
in particular, finance.

QUESTIONS?
COMMENTS?