PROBLEMS IN PRACTICE TEST 4

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52. Consider the following system of linear equations over the real numbers, where x, y and z are variables and b is a real constant.

$$\begin{array}{ccccccc} x & + & y & + & z & = & 0 \\ x & + & 2y & + & 3z & = & 0 \\ x & + & 3y & + & bz & = & 0 \end{array}$$

Which of the following statements are true?

I. There exists a value of b for which the system has no solution.
II. There exists a value of b for which the system has exactly one solution.
III. There exists a value of b for which the system has more than one solution.

(A) II only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III

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53. In the complex plane, let C be the circle $|z| = 2$ with positive (counterclockwise) orientation. Then $\displaystyle\int_C \frac{dz}{(z-1)(z+3)^2} =$

(A) 0
(B) $2\pi i$
(C) $\dfrac{\pi i}{2}$
(D) $\dfrac{\pi i}{8}$
(E) $\dfrac{\pi i}{16}$

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54. The inside of a certain water tank is a cube measuring 10 feet on each edge and having vertical sides and no top. Let $h(t)$ denote the water level, in feet, above the floor of the tank at time t seconds. Starting at time $t = 0$, water pours into the tank at a constant rate of 1 cubic foot per second, and, simultaneously, water is removed from the tank at $0.25\,[h(t)]$ cubic feet per second. As $t \to \infty$, what is the limit of the volume of the water in the tank?

(A) 400 cubic feet
(B) 600 cubic feet
(C) $1,000$ cubic feet
(D) The limit does not exist.
(E) The limit exists, but it cannot be determined without knowing $h(0)$.

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55. Suppose that f is a twice-differentiable function on the set of real numbers and that $f(0)$, $f'(0)$ and $f''(0)$ are all negative. Suppose f'' has all three of the following properties:

I. It is increasing on the interval $[0, \infty)$.
II. It has a unique zero in the interval $[0, \infty)$.
III. It is unbounded on the interval $[0, \infty)$.

Which of the same three properties does f necessarily have?

(A) I only
(B) II only
(C) III only
(D) II and III only
(E) I, II and III

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56. For every nonempty set S and every metric d on S, which of the following is a metric on S?

(A) $4 + d$
(B) $e^d - 1$
(C) $d - |d|$
(D) d^2
(E) $\sqrt{d}$

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57. Let $\mathbb{R}$ be the field of real numbers and $\mathbb{R}[x]$ the ring of polynomials in x with coefficients in $\mathbb{R}$. Which of the following subsets of $\mathbb{R}[x]$ is a subring of $\mathbb{R}[x]$?

I. All polynomials in $\mathbb{R}[x]$ whose coefficient of x is zero
II. All polynomials in $\mathbb{R}[x]$ whose degree is an even integer, together with the zero polynomial
III. All polynomials in $\mathbb{R}[x]$ whose coefficients are rational numbers

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) I, II and III

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58. Let f be a real-valued function defined and continuous on the set $\mathbb{R}$ of real numbers. Which of the following must be true of the set $S := \{f(c) \mid 0 < c < 1\}$?

I. S is a connected subset of $\mathbb{R}$.
II. S is an open subset of $\mathbb{R}$.
III. S is a bounded subset of $\mathbb{R}$.

(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III
