MATH 1271 Spring 2012, Midterm #2
Handout date: Thursday 29 March 2012

PRINT YOUR NAME:

SOLUTIONS
Versión A

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I understand the above, and I understand that cheating has severe consequences, from a failing grade to expulsion.

SIGN YOUR NAME:

I. Multiple choice

A. (5 pts) (no partial credit) Find the logarithmic derivative of $(2+x^4)^{\cos x}$ w.r.t. $x$.

(a) $(-\sin x)(4x^3/(2+x^4))$

(b) $(\cos x)(\ln(2+x^4)) + (-\sin x)(4x^3/(2+x^4))$

(c) $(\cos x)(\ln(2+x^4))$

(d) $(-\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))$

(e) NONE OF THE ABOVE

$\frac{d}{dx}\left[(\cos x)(\ln(2+x^4))\right]$

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B. (5 pts) (no partial credit) Find the derivative of $(2+x^4)^{\cos x}$ w.r.t. $x$.

(a) $[(2+x^4)^{\cos x}][(-\sin x)(4x^3/(2+x^4))]$

(b) $[(2+x^4)^{\cos x}][(\cos x)(\ln(2+x^4)) + (-\sin x)(4x^3/(2+x^4))]$

(c) $[(2+x^4)^{\cos x}][(\cos x)(\ln(2+x^4))]$

(d) $[(2+x^4)^{\cos x}][(-\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))]$

(e) NONE OF THE ABOVE

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$-(x^2-4x+3) = -(x-1)(x-3)$

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C. (5 pts) (no partial credit) Suppose $f''(x) = -x^2+4x-3$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

(a) $f$ is concave down on $(-\infty, 1]$, up on $[1,3]$ and down on $[3,\infty)$.

(b) $f$ is concave up on $(-\infty, 1]$, down on $[1,3]$ and up on $[3,\infty)$.

(c) $f$ is concave down on $(-\infty, -3]$, up on $[-3,-1]$ and down on $[-1,\infty)$.

(d) $f$ is concave up on $(-\infty, -3]$, down on $[-3,-1]$ and up on $[-1,\infty)$.

(e) NONE OF THE ABOVE

$f''$ neg 0 pos 0 neg

1 3

D. (5 pts) (no partial credit) Find an equation of the tangent line to $4x^2y - 2y^3 = 2$ at the point $(1, 1)$.

(a) $y - 1 = 4(x - 1)$

(b) $y - 1 = 3(x - 1)$

(c) $y - 1 = 2(x - 1)$

(d) $y - 1 = x - 1$

(e) NONE OF THE ABOVE

$$8xy + 4x^2y' - 6y^2y' = 0$$

$$y' = \frac{-8xy}{4x^2 - 6y^2}$$

$$\text{slope} = \frac{-8}{4-6} = \frac{-8}{-2} = 4$$

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E. (5 pts) (no partial credit) Compute $[d/dx][\sin(\cos(e^x + 3))]$.

(a) $\cos(\cos(e^x + 3))$

(b) $[\cos(\cos(e^x + 3))][-\sin(e^x + 3)][e^x + 3]$

(c) $[\cos(\cos(e^x + 3))][\cos(e^x + 3)][e^x + 3]$

(d) $0$

(e) NONE OF THE ABOVE

$$= [\cos(\cos(e^x+3))][-\sin(e^x+3)][e^x]$$

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F. (5 pts) (no partial credit) Find the logarithmic derivative of $x^2 + 7x - 8$ w.r.t. $x$.

(a) $\dfrac{x^2 + 7x - 8}{2x + 7}$

(b) $\ln(2x + 7)$

(c) $\dfrac{2x + 7}{x^2 + 7x - 8}$

(d) $(\ln(x^2)) + 7(\ln x) - (\ln 8)$

(e) NONE OF THE ABOVE

II. True or false (no partial credit):

a. (5 pts) If $f' = g'$ on an interval $I$, then $f - g$ is constant on $I$.

True

b. (5 pts) Every critical number occurs at local extremum.

False

c. (5 pts) If $f'(7) = 0$ and $f''(7) < 0$, then $f$ has a local maximum at 7.

True

d. (5 pts) If $f'' > 0$ on an interval $I$, then $f$ is concave up on $I$.

True

e. (5 pts) Assume that $\lim_{x\to a}[f(x)] = 0 = \lim_{x\to a}[g(x)]$. Assume also that $\lim_{x\to a}\frac{f'(x)}{g'(x)} = 7$. Then $\lim_{x\to a}\frac{f(x)}{g(x)} = 7$.

True

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION A

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1ab.

III. 2.

III. 3,4.

III. 5.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. a. (5 pts) Compute $\frac{d}{dx}\left[\frac{2x^3-8}{3+(\arctan(2x))}\right]$.

$$\| $$

$$\frac{[3+(\arctan(2x))][6x^2]-[2x^3-8]\left[\frac{1}{1+(2x)^2}\right][2]}{[3+(\arctan(2x))]^2}$$

b. (5 pts) Compute $\frac{d}{dx}[(4-\sin x)^x]$.

$$\| $$

$$\left[(4-\sin x)^x\right]\left[\frac{d}{dx}\left[x(\ln(4-\sin x))\right]\right]$$

$$\| $$

$$\left[(4-\sin x)^x\right]\left[(\ln(4-\sin x))+x\left(\frac{-\cos x}{4-\sin x}\right)\right]$$

2. (10 pts) Using implicit differentiation, find $y' = dy/dx$, assuming that $(x - y^2)^5 = x$.

$$5(x-y^2)^4(1-2yy') = 1$$

$$\left[5(x-y^2)^4\right] - \left[10y(x-y^2)^4\right]y' = 1$$

$$y' = \frac{1 - 5(x-y^2)^4}{-10y(x-y^2)^4}$$

3. (5 pts) Let $f(x) = 2x + 6x^5$. Then $f$ is a one-to-one function. Let $g := f^{-1}$. Then $f(1) = 8$, so $g(8) = 1$. Compute $g'(8)$.

$$g'(8) = \frac{1}{f'(1)} = \frac{1}{\left[2+30x^4\right]_{x\mapsto 1}} = \frac{1}{32}$$

4. (10 pts) Find the maximal intervals of concavity for $f(x) = -3x^5 + 20x^4 + 7x + 3$. For each interval, state clearly whether $f$ is concave up or concave down on that interval.

$$f'(x) = -15x^4 + 80x^3 + 7$$

$$f''(x) = -60x^3 + 240x^2 = -60x^2(x-4)$$

| $f''$ | pos | $0^2$ | pos | $0$ | neg |
|---|---|---|---|---|---|
| | | $0$ | | $4$ | |

$f$ is concave up on $(-\infty, 4]$.
$f$ is concave down on $[4, \infty)$.

5. (10 pts) Compute $\lim_{x\to 1}\left[\frac{\ln x}{\cos(\pi x/2)}\right]$.

$$\overset{\text{l'H } \frac{0}{0}}{=}$$

$$\lim_{x\to 1}\left[\frac{1/x}{[-\sin(\pi x/2)][\pi/2]}\right]$$

$$=$$

$$\frac{1/1}{[-\sin(\pi/2)][\pi/2]}$$

$$=$$

$$\frac{1}{[-1][\pi/2]}$$

$$=$$

$$-\frac{2}{\pi}$$