MATH 1271 Spring 2014, Midterm #2
Handout date: Thursday 17 April 2014
Instructor: Scot Adams

PRINT YOUR NAME:

SOLUTIONS
Version D

PRINT YOUR X.500 ID:

PRINT YOUR TA'S NAME:

WHAT RECITATION SECTION ARE YOU IN?

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind. Turn off all handheld devices, including cell phones.

Show work; a correct answer, by itself, may be insufficient for credit. Arithmetic need not be simplified, unless the problem requests it.

I. Multiple choice

A. (5 pts) (no partial credit) Let $f(x) = e^{2x} + 3x$. What is the iterative formula of Newton's method used to solve $f(x) = 0$? Circle one of the following answers:

(a) $x_{n+1} = x_n - \dfrac{e^{2x_n} + 3x_n}{e^{2x_n} + 3}$

(b) $x_{n+1} = x_n - \dfrac{2e^{2x_n} + 3}{e^{2x_n} + 3x_n}$

(c) $x_{n+1} = x_n - \dfrac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3}$

(d) $x_{n+1} = x_n - \dfrac{e^{2x_n} + 3}{e^{2x_n} + 3x_n}$

(e) NONE OF THE ABOVE

$$x_{n+1} = x_n - \frac{e^{2x_n} + 3x_n}{2e^{2x_n} + 3}$$

B. (5 pts) (no partial credit) Let $y = x^2 + x$. Compute dy, evaluated at $x = 10$, $dx = 0.1$. Circle one of the following answers:

$$dy = (2x+1)\,dx$$

(a) 1.2

(b) 2.1

(c) 1.22

(d) 2.11

(e) NONE OF THE ABOVE

$$(2\cdot 10 + 1)(0.1) = (21)(0.1) = 2.1$$

C. (5 pts) (no partial credit) Find the derivative of $(2+x^4)^{\sin x}$ w.r.t. x. Circle one of the following answers:

(a) $[(2+x^4)^{\sin x}][(\sin x)(\ln(2+x^4))]$

(b) $[(2+x^4)^{\sin x}][(\cos x)(4x^3/(2+x^4))]$

(c) $[(2+x^4)^{\sin x}][(\sin x)(\ln(2+x^4)) + (\cos x)(4x^3/(2+x^4))]$

(d) $[(2+x^4)^{\sin x}][(\cos x)(\ln(2+x^4)) + (\sin x)(4x^3/(2+x^4))]$

(e) NONE OF THE ABOVE

LD: $[(2+x^4)^{\sin x}]\left[\dfrac{d}{dx}\left[(\sin x)(\ln(2+x^4))\right]\right]$

D. (5 pts) (no partial credit) Let $f(x) = \cot^2(5x^4 + 1)$. Compute $\int_5^5 f(x)\,dx$. Circle one of the following answers:

(a) 20

(b) 6

(c) 2

(d) 0

(e) NONE OF THE ABOVE

E. (5 pts) (no partial credit) Suppose $f''(x) = (x-7)^2(x-8)^4$. At most one of the following statements is true. If one is, circle it. Otherwise, circle "NONE OF THE ABOVE".

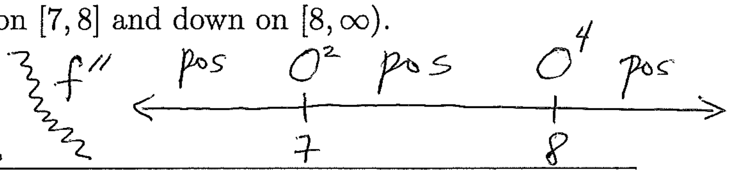
(a) f is concave down on $(-\infty, 7]$ and up on $[7, \infty)]$.

(b) f is concave up on $(-\infty, 7]$ and down on $[7, \infty)]$.

(c) f is concave up on $(-\infty, 7]$, down on $[7, 8]$ and up on $[8, \infty)$.

(d) f is concave down on $(-\infty, 7]$, up on $[7, 8]$ and down on $[8, \infty)$.

(e) NONE OF THE ABOVE



f is concave up on $\mathbb{R} = (-\infty, \infty)$.

$= 2e^2 + 2e^8$

F. (5 pts) (no partial credit) Let $f(x) = e^{3x-4}$. Recall that $M_2S_1^5 f$ denotes the midpoint Riemann sum, from 1 to 5, of f, with two subintervals. Which of these is equal to $M_2S_1^5 f$? Circle one of the following answers:

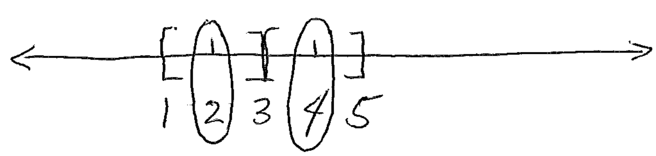
(a) $2(e^2 + e^8)$

(b) $e^2 + e^8$

(c) $2(e^5 + e^{11})$

(d) $e^5 + e^{11}$

(e) NONE OF THE ABOVE

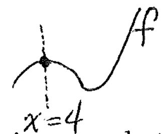


$f(2) = e^{6-4} = e^2$

$f(4) = e^{12-4} = e^8$

II. True or false (no partial credit):

a. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be any function such that $f'(4) = 0$ and $f''(4) < 0$. Assume that f'' is defined on $\mathbb{R}$. Then f has a global maximum at 4.



False

b. (5 pts) Let $f, g : \mathbb{R} \to \mathbb{R}$ be any two differentiable functions such that, for all $x \in \mathbb{R}$, $f'(x) = g'(x)$. Then $f - g$ is a constant.

MVT

True

c. (5 pts) Assume that $\lim_{x\to a}[f(x)] = 0 = \lim_{x\to a}[g(x)]$. Assume also that $\lim_{x\to a} \frac{f'(x)}{g'(x)} = -\infty$. Then $\lim_{x\to a} \frac{f(x)}{g(x)} = -\infty$.

$l'H \frac{0}{0}$

True

d. (5 pts) $\underbrace{\frac{d}{dx}\left[\int_1^x \sin(e^t)\,dt\right]}_{= \sin(e^x)} = \cos(e^x)$.

FTC

False

e. (5 pts) If f is continuous on $[a, b]$, then $\int_a^b (f(x))\,dx = \lim_{n\to\infty} [R_n S_a^b f]$.

Definition of $\int_a^b (f(x))\,dx$

True

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PLEASE DO NOT WRITE BELOW THE LINE

VERSION D

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1.

III. 2.

III. 3.

III. 4.

III. Computations. Show work. Unless otherwise specified, answers must be exactly correct, but can be left in any form easily calculated on a standard calculator.

1. (10 pts) Find an antiderivative w.r.t. x of $\cos^2(2x+3)$. (Hint: $\cos(2\theta) = -1+2(\cos^2\theta)$.)

$$1 + [\cos(2\theta)] = 2(\cos^2\theta)$$

$$\frac{1}{2} + \frac{1}{2}[\cos(2\theta)] = \cos^2\theta$$

$$\theta :\longrightarrow 2x+3$$

$$\frac{1}{2} + \frac{1}{2}[\cos(4x+6)] = \cos^2(2x+3)$$

Antiderivative:

$$\frac{1}{2}x + \frac{1}{2}\left[\frac{\sin(4x+6)}{4}\right]$$

2. (10 pts) Let $f(x) = \int_{2+5x}^{1+e^x} \underbrace{\sqrt{t^3+1}}_{H'(t)}\, dt$. Compute $f'(0)$.

$$f(x) \overset{FTC}{=\!=} \left(H(1+e^x)\right) - \left(H(2+5x)\right)$$

$$f'(x) \overset{CR}{=\!=} \left(H'(1+e^x)\right)(e^x) - \left(H'(2+5x)\right)(5)$$

$$f'(0) = \left(H'(2)\right)(1) - \left(H'(2)\right)(5)$$

$$= \left(H'(2)\right)(1-5) = \left(H'(2)\right)(-4)$$

$$= \left(\sqrt{8+1}\right)(-4) = \left(\sqrt{9}\right)(-4)$$

$$= -12$$

3. (15 pts) We are asked to design a large cup in the shape of a cylinder. The cup is to have an open top, and must contain 2π cubic feet of volume inside. Let r be the radius of the top of the cup. On the interval $r > 0$, find the choice of r (in feet) that minimizes the surface area, A, of the cup. (HINT: Our local precalculus expert shows us the formula that relates A to r. It is $A = \pi r^2 + (4\pi/r)$.)

$$\frac{dA}{dr} = 2\pi r - \frac{4\pi}{r^2} = \frac{2\pi r^3 - 4\pi}{r^2}$$

$$= \frac{2\pi (r^3 - 2)}{r^2}$$

dA/dr		neg	0	pos
r	0		$\sqrt[3]{2}$	

On $r > 0$, A attains a global minimum only at $r = \sqrt[3]{2}$ ft.

4. (10 pts) A conical pile of sand is growing. Its height is always equal to the radius, r, of its base. Assume that its volume is always growing at a rate of 5π cubic feet per minute. Find the rate of growth in r (in feet per minute) at the moment when the volume is 9π cubic feet. (HINT: According to our local precalculus expert, its volume, V, is given by $V = \pi r^3/3$.)

t_0

$$* := [t :\to t_0]$$

$$? := \dot{r}_*$$

$$9\pi = V_* = \pi r_*^3/3$$

$$27\cancel{\pi} = \cancel{\pi} r_*^3$$

$$3 = r_*$$

$$5\pi = \dot{V} = \pi(\cancel{3}r^2\dot{r})/\cancel{3} = \pi r^2 \dot{r}$$

$$5\pi = \pi r_*^2 \dot{r}_* = \pi(9)(?)$$

$$? = \frac{5\cancel{\pi}}{9\cancel{\pi}} = \frac{5}{9} \text{ ft/min}$$