CALCULUS

Derivatives and rates of change

OLD2

WARNING: In this homework, derivatives must be computed from the defintion, *i.e.*, as the limit of the difference quotient. Do NOT use product, quotient or chain rules, or any other technique coming from a later topic.

0270-1. Let C be the curve $y = x^2 - 3x + 5$.
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Let L be the tangent line to C at the point $(1, 3)$.

a. Find the slope of L, by computing a limit of slopes of secant lines.

b. Find an equation of L.

c. Graph C and L in the rectangle

$$-1 \le x \le 4, \qquad -1 \le y \le 6.$$

d. Graph C and L in the rectangle

$$0 \le x \le 2, \qquad 2 \le y \le 6.$$

e. Graph C and L in the rectangle

$$0.9 \le x \le 1.1, \qquad 2.8 \le y \le 3.2.$$

In c, d and e, note that, as you "zoom in", the tangent line looks more and more like the curve.

0270-2. OLD2

a. Compute $\lim_{h \to 0} \frac{\sqrt{9+h}-3}{h}$.

b. Find the slope of the secant line to $y = \sqrt{x+1}$ through the points

$(8, 3)$ and $(8+h, \sqrt{9+h})$.

c. Find an equation of the tangent line to $y = \sqrt{x+1}$ at the point $(8, 3)$.

0270-3. OLD2

A particle moves on a number line. Its position at any time t is $\sqrt{t+1}$.

a. Find the average velocity between time $t = 8$ and time $t = 8 + h$.

b. Find the instantaneous velocity at time $t = 8$.

0270-4. A heavy object is taken to the top of a building 150 feet high. At time $t = 0$, it is thrown upward at 25 feet/second. We engage the services of two Nobel prize-winning physicists who confer (*i.e.*, yell and scream at one another). After several hours of scholarly study, followed by minor medical treatment for blunt trauma, lacerations and contusions, they hold a joint press conference, and inform their public that, t seconds after release, the object will be located

$$150 + 25t - 16t^2 \quad \text{feet}$$

above the ground. Based on this, find the the velocity of the object 0.3 seconds after release. Give your answer in feet per second.

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0270-5.
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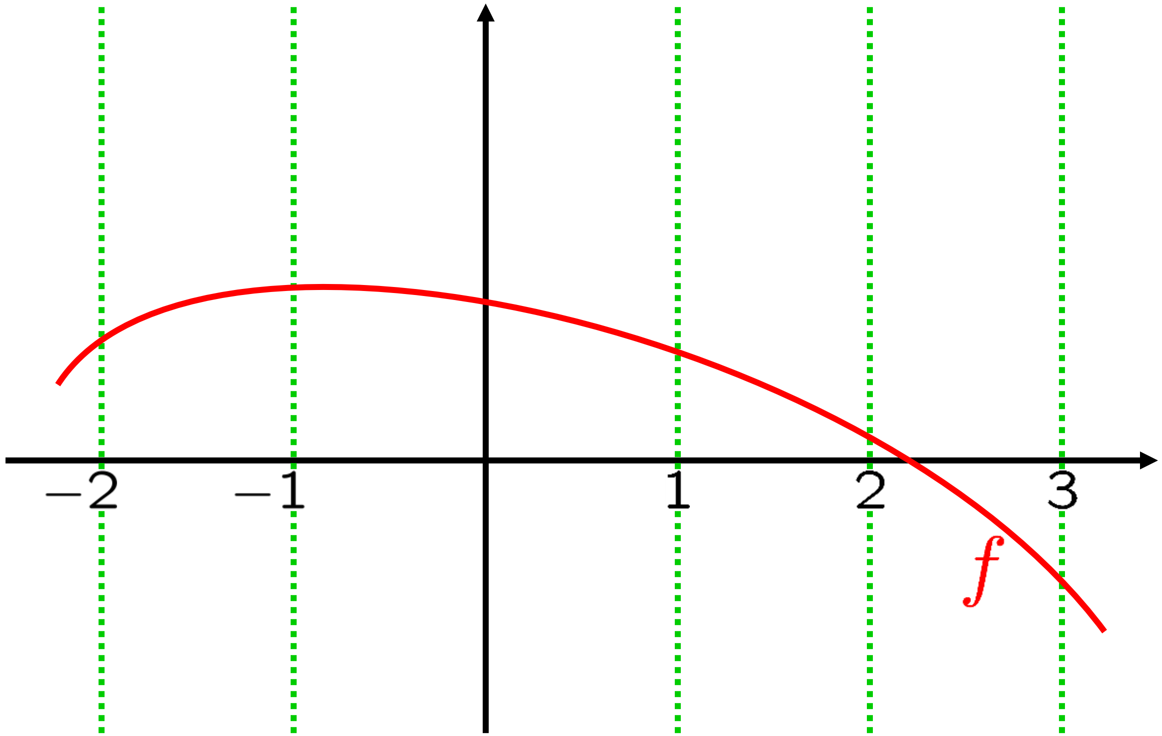

Order these numbers, from smallest to largest:

$$f'(-2),\ f'(-1),\ f'(0),\ f'(1),\ f'(2),\ f'(3)$$

Note that we are asking about f', not f.

0270-6. Let $f(x) = \dfrac{4x-2}{3x+7}$.

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Do NOT use the quotient rule.
Use only the definition of the derivative as the limit of the difference quotient.

a. Compute $f'(2)$.

b. Compute $f'(3)$.

c. Compute $f'(4)$.

d. Compute $f'(a)$, for an arbitrary number a.

0270-7. Find a function f and a number a s.t.
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$$f'(a) = \lim_{h \to 0} \frac{[\cot^2(-7+h)] - [\cot^2(-7)]}{h}.$$