CALCULUS
Volume by slices and
the disk and washer methods:
Problems
OLD2

0720-1. Let R be the region bounded by
$$y = x + 1 \text{ and } x = 3 \text{ in } 2 \le y \le 3.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

0720-2. Let R be the region bounded by
$$y = x^2 \text{ and } y = 2x + 3.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the line $x = -2$.

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0720-3. Let R be the region bounded by
$$y = \ln x, \ x = 9 \text{ and } y = 2.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the y-axis.

0720-4. Let R be the region bounded by
$$y = \sin x \text{ and } y = 0 \text{ in } 0 \le x \le \frac{\pi}{3}.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

$$\text{Hint: } \sin^2 x = \frac{1 - [\cos(2x)]}{2}$$

Let R be the region bounded by

$$(x-1)^2 + (y-3)^2 = 4.$$

  a. Sketch R.
  b. Find the volume of the solid obtained by
        rotating R about the x-axis.

Note: This solid is called a torus. It is in the
          shape of a doughnut.

Hint: Remember that $2\displaystyle\int_{-2}^{2}\sqrt{4-u^2}\,du$ is known;

    it is the area enclosed in a circle of radius 2.

0720-6. Let R be the region bounded by
$$y = x^2 \text{ and } x = y^4.$$
   a. Sketch R.
   b. Find the volume of the solid obtained by rotating R about the line $y = -1/3$.
   c. Find the volume of the solid obtained by rotating R about the line $x = -1/2$.

0720-7. Let R be the region bounded by
$$y = x^3 \text{ and } x = y^6.$$
   a. Sketch R.
   b. Find the volume of the solid obtained by rotating R about the line $y = -1/3$.
   c. Find the volume of the solid obtained by rotating R about the line $x = -1/2$.

0720-8. Let R be the region bounded by
$y = -\sin x$, $y = e^x$ in $0 \le x \le \pi/3$.

Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating R about the line $y = -3$.

0720-9. Describe the solid of revolution whose volume is given by

$$\pi \int_{1/2}^{3/2} \left(9e^{8x} - 4\cos^2 x \right) dx.$$

Do not evaluate this integral.

0720-10. Describe the solid of revolution whose volume is given by

$$\pi \int_{\pi/2}^{\pi} (3 + \cos x)^2 - 9 \, dx.$$

Do not evaluate this integral.

A solid S sits above a horizontal plane P. $\forall x \geq 0$, let P_x be the horizontal plane that is x units above P. Suppose that S lies between P_1 and P_2. Suppose, also, that $\forall x \in [1, 2]$, the intersection of S and P_x is the region inside a triangle

whose base has length $5x$

and whose altitude has length e^{3x^2}.

Compute the volume of S.

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0720-12. Using the disk method, find the volume in a ball of radius 37, following the diagram shown below.

disk

ball

37

−37

37
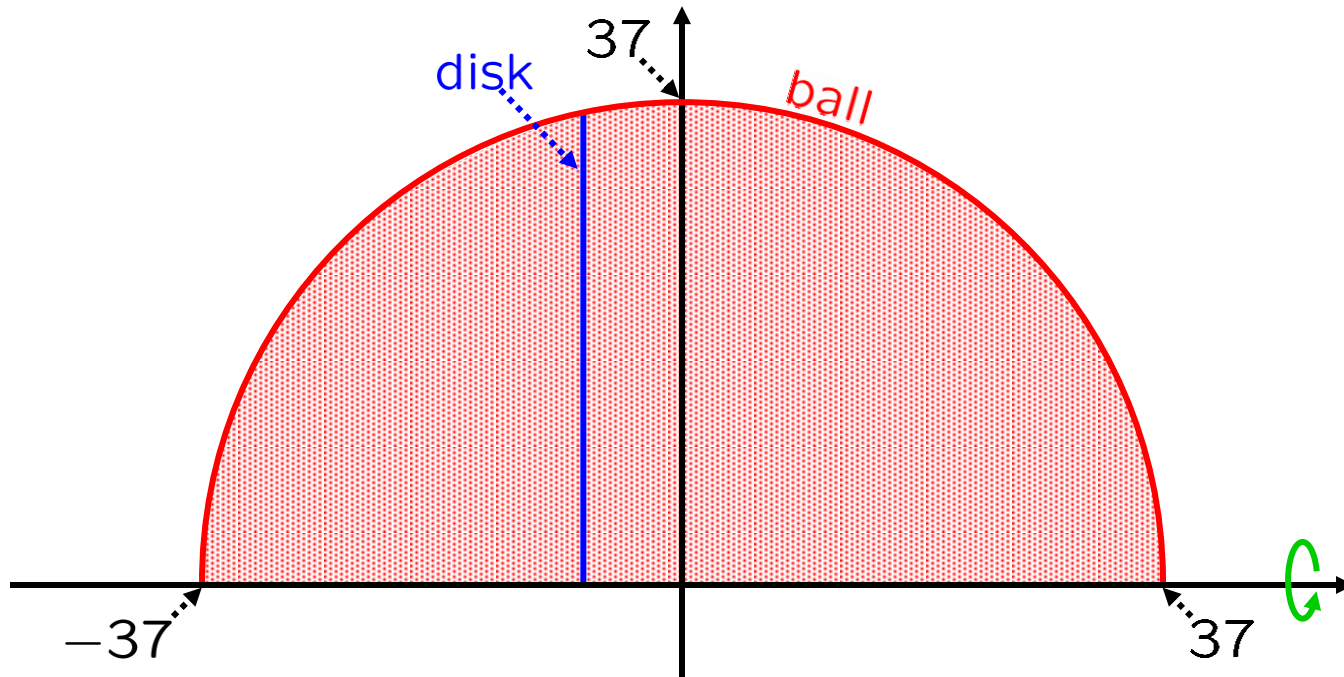
We create a napkin holder by drilling a cylindrical hole of radius 12 through the middle of a ball of radius 37, as shown below. Using the washer method, find its volume.

OLD2

37

washer

napkin holder

12

−37

37

9