CALCULUS
Volume by slices and the disk and washer methods: Problems

OLD

0720-1. Let R be the region bounded by
OLD

$y = x + 1$ and $x = 2$ in $1 \leq y \leq 2$.

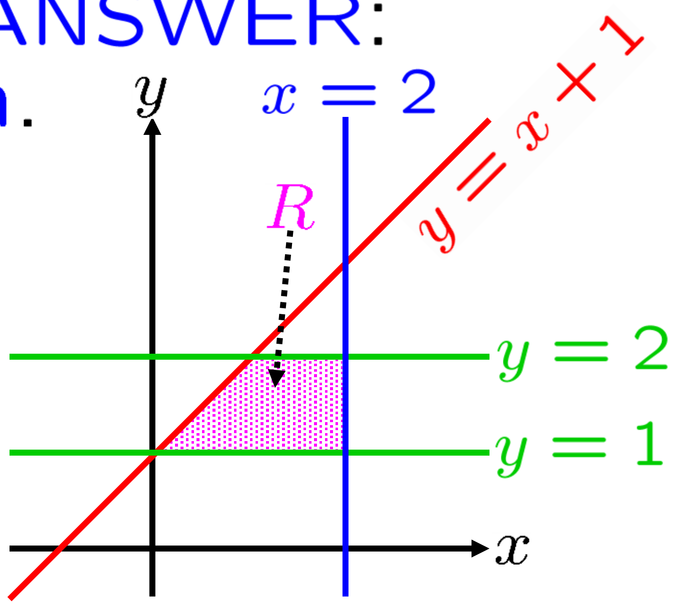
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

ANSWER:

a.



0720-1. Let R be the region bounded by
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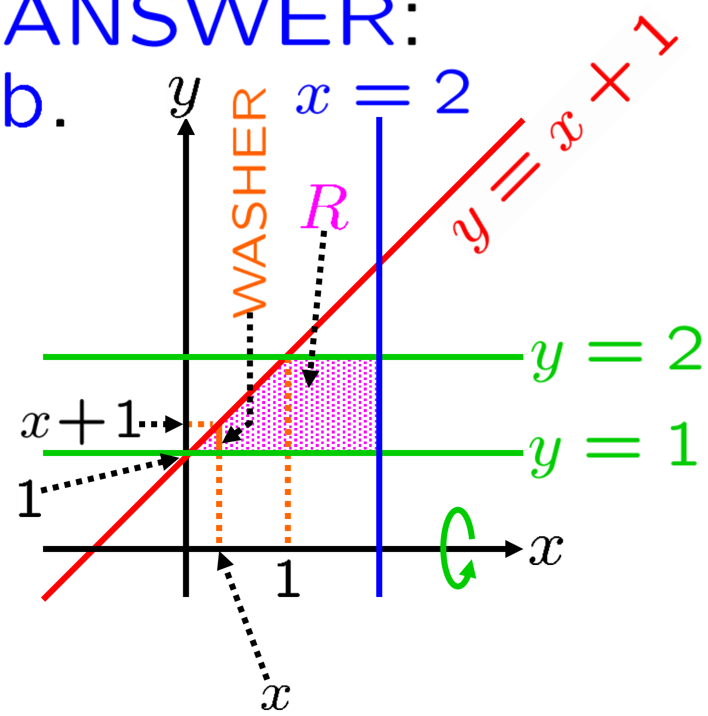
ANSWER:

b.



$$\int_0^1 [\pi(x+1)^2 - \pi(1^2)]\,dx \quad +$$

continued on next slide

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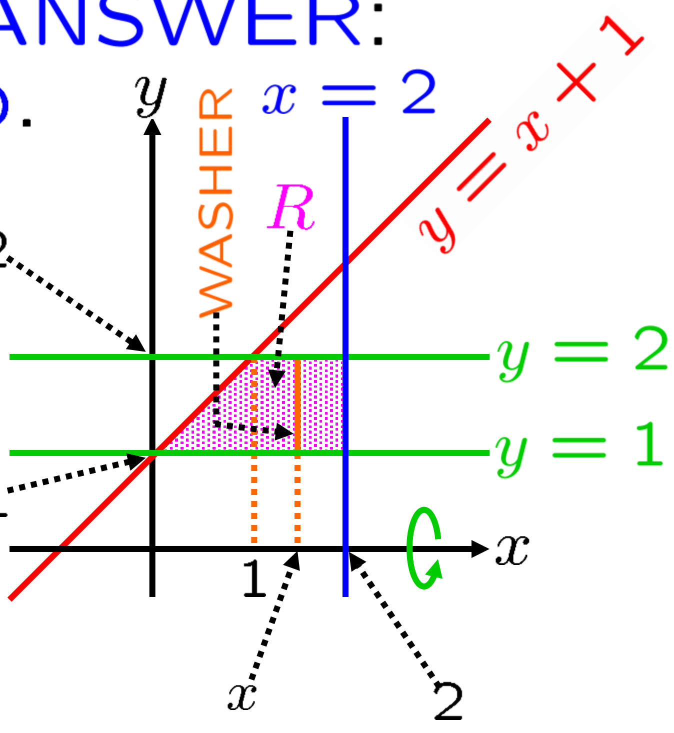
ANSWER:

b.



$$\int_0^1 [\pi(x+1)^2 - \pi(1^2)]\,dx \quad + \int_1^2 \pi(2^2) - \pi(1^2)\,dx$$

0720-1. Let R be the region bounded by
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ANSWER:

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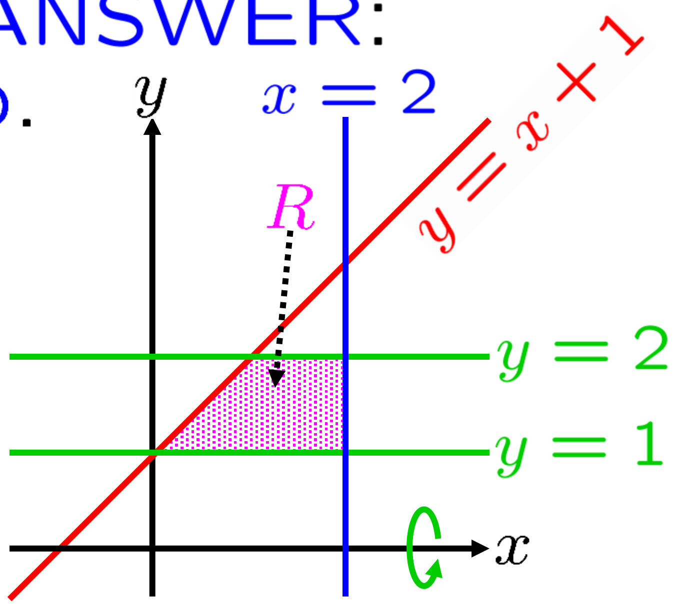

$$\int_0^1 [\pi(x+1)^2 - \pi(1^2)]\,dx \quad + \int_1^2 \pi(2^2) - \pi(1^2)\,dx$$

$$= \pi\left[\int_0^1 x^2 + 2x\,dx\right] + \pi\left[\int_1^2 3\,dx\right]$$

$$= \pi\left[\frac{1^3}{3} + 1^2\right] + \pi\,[3]$$

$$= \pi\left[\frac{1}{3} + 1 + 3\right] = \frac{13\pi}{3}$$

0720-1. Let R be the region bounded by $y = x + 1$ and $x = 2$ in $1 \le y \le 2$.

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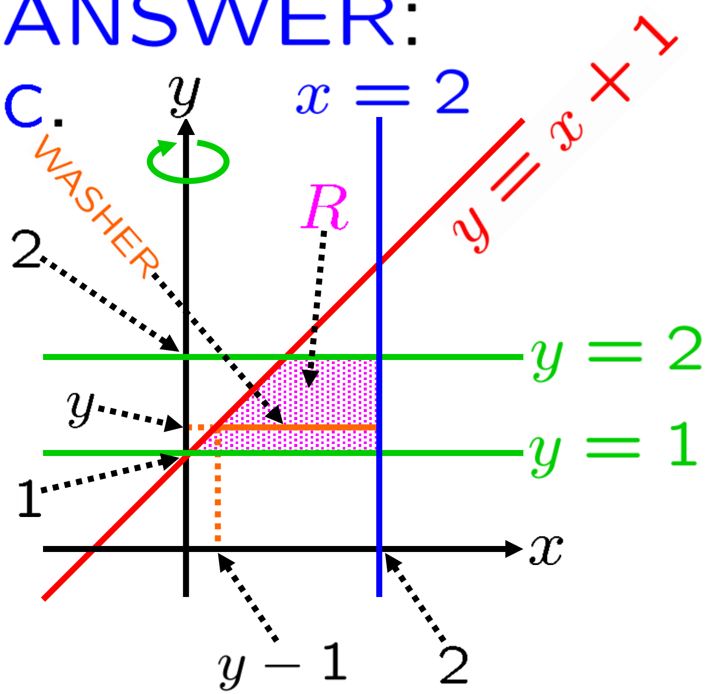
ANSWER:

c.



$$\int_1^2 [\pi(2^2) - \pi(y-1)^2]\,dy$$

$$= \pi \int_1^2 4 - (y^2 - 2y + 1)\,dy$$

$$= \pi \int_1^2 -y^2 + 2y + 3\,dy$$

$$= \pi \left[-\left(\frac{2^3 - 1^3}{3} \right) + (2^2 - 1^2) + 3(2-1) \right]$$

$$= \pi \left[-\frac{7}{3} + 6 \right] = \frac{11\pi}{3} \blacksquare$$

0720-2. Let R be the region bounded by
OLD

$$y-1=(x-1)^2 \text{ and } y=x.$$

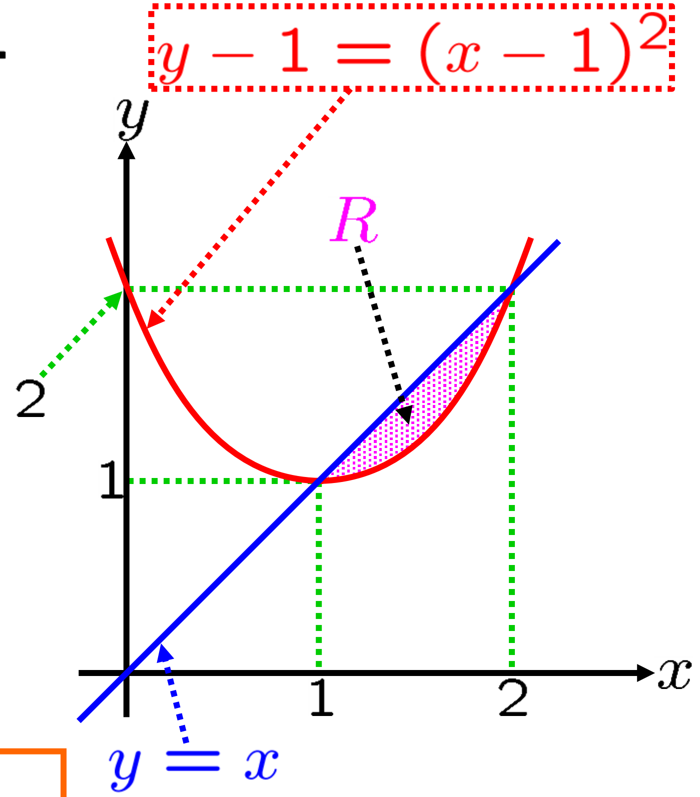
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

ANSWER:

a.



0720-2. Let R be the region bounded by

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$$y-1=(x-1)^2 \text{ and } y=x.$$

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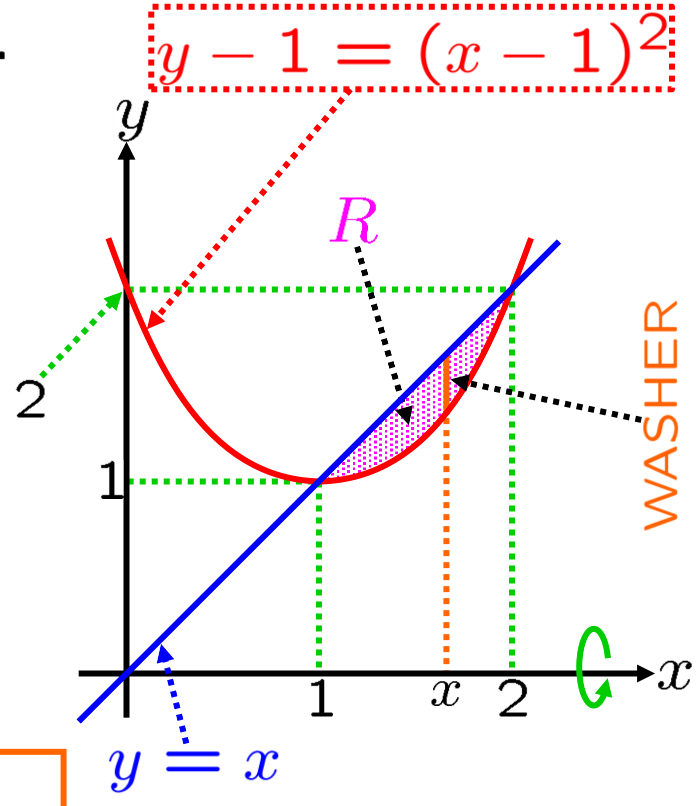
ANSWER:

b.



$$\int_1^2 \pi x^2 - \pi\left(1+(x-1)^2\right)^2\,dx$$

0720-2. Let R be the region bounded by

$$y - 1 = (x-1)^2 \text{ and } y = x.$$

OLD

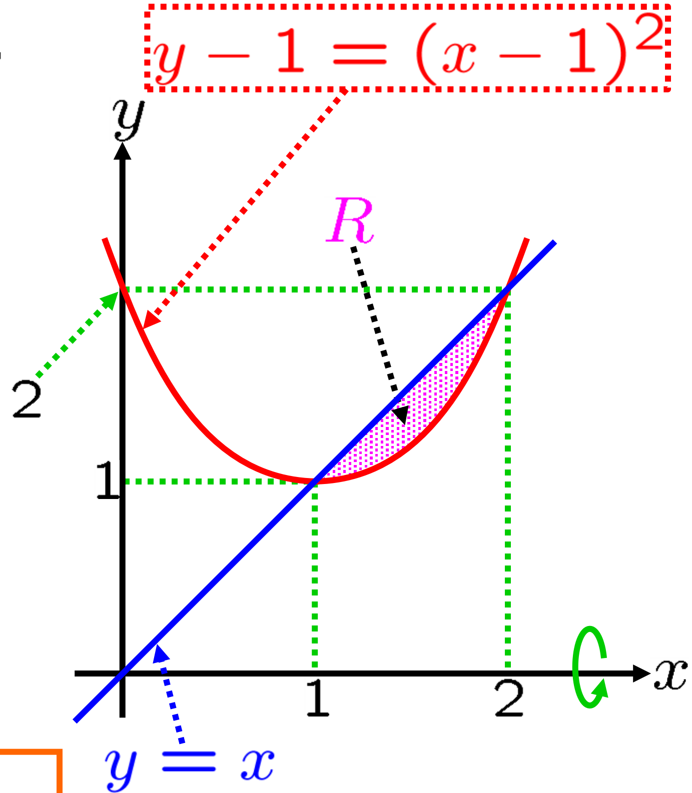
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

ANSWER:

b.



$$\int_1^2 \pi x^2 - \pi\left(1 + (x-1)^2\right)^2 \, dx$$

$$= \pi \int_1^2 x^2 - \left(x^2 - 2x + 2\right)^2 \, dx$$

$$= \pi \int_1^2 x^2 - \left(x^4 - 4x^3 + 8x^2 - 8x + 4\right) \, dx$$

$$= \pi \int_1^2 -x^4 + 4x^3 - 7x^2 + 8x - 4 \, dx$$

$$= \pi \left[-\frac{31}{5} + 15 - \frac{49}{3} + 12 - 4\right]$$

$$= \pi \left[-\frac{93}{15} + 23 - \frac{245}{15}\right]$$

0720-2. OLD Let R be the region bounded by $y-1=(x-1)^2$ and $y=x$.

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

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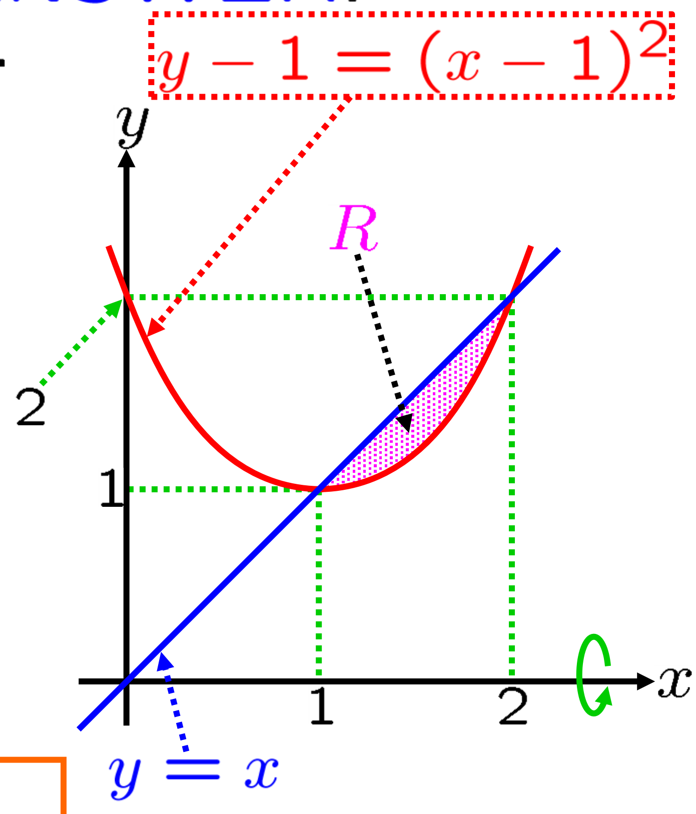
ANSWER:

b.



$$\int_1^2 \pi x^2 - \pi\left(1+(x-1)^2\right)^2\,dx$$

$$= \pi\left[-\frac{93}{15}+23-\frac{245}{15}\right]$$

$$= \pi\left[-\frac{93}{15}+\frac{345}{15}-\frac{245}{15}\right]$$

$$= \frac{7\pi}{15}$$

0720-2. OLD Let R be the region bounded by

$$y-1=(x-1)^2 \text{ and } y=x.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

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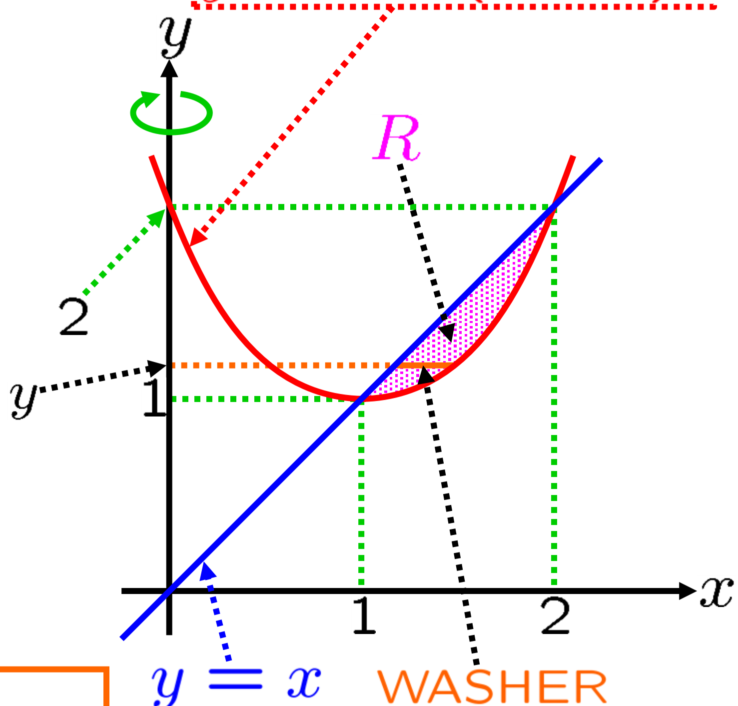
ANSWER:

c. $y-1=(x-1)^2$



$$\int_1^2 \pi\left(1+\sqrt{y-1}\right)^2-\pi\left(y^2\right)\,dy$$

0720-2. Let R be the region bounded by

OLD

$$y-1=(x-1)^2 \text{ and } y=x.$$

a. Sketch R.

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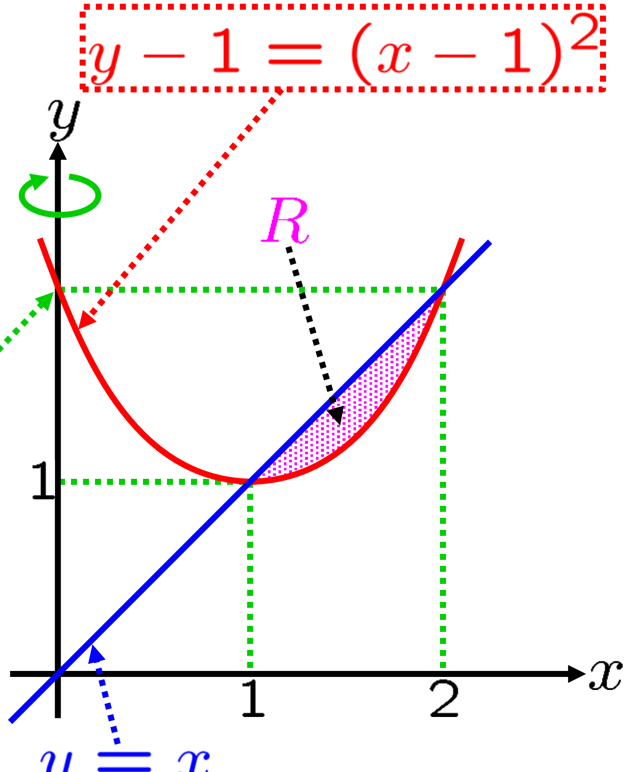
ANSWER:

c.



$$\int_1^2 \pi\left(1+\sqrt{y-1}\right)^2-\pi\left(y^2\right)\,dy$$

$$=\pi\int_1^2 \left(\not{1}+2\sqrt{y-1}+y-\not{1}\right)-y^2\,dy$$

$$=\pi\int_1^2 -y^2+y+2(y-1)^{1/2}\,dy$$

0720-2. Let R be the region bounded by
OLD

$$y - 1 = (x-1)^2 \text{ and } y = x.$$

a. Sketch R.

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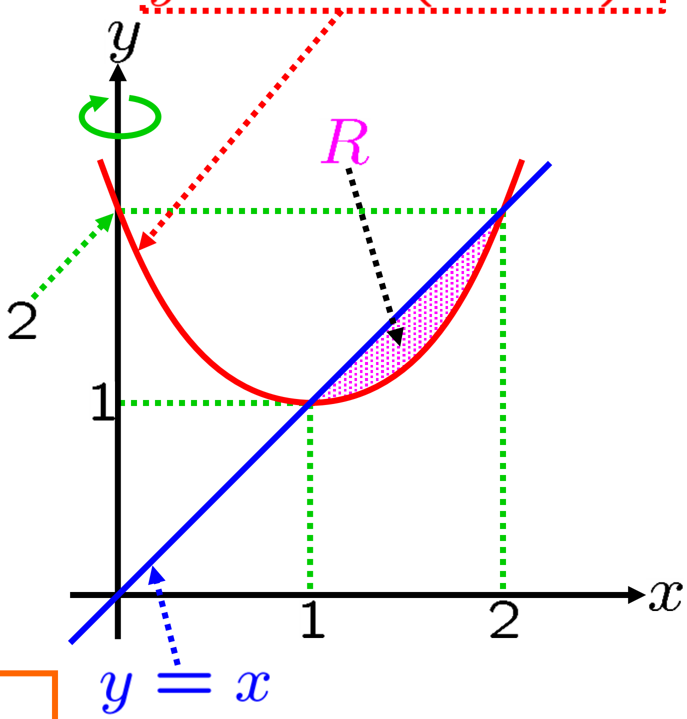
ANSWER:

c.



$$\int_1^2 \pi\left(1+\sqrt{y-1}\right)^2 - \pi\left(y^2\right)\, dy$$

$$= \pi \int_1^2 -y^2 + y + 2(y-1)^{1/2}\, dy$$

$$= \pi\left[-\frac{y^3}{3} + \frac{y^2}{2} + \frac{2(y-1)^{3/2}}{3/2}\right]_{y:\to 1}^{y:\to 2}$$

0720-2. Let R be the region bounded by

$$y-1=(x-1)^2 \text{ and } y=x.$$

OLD

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

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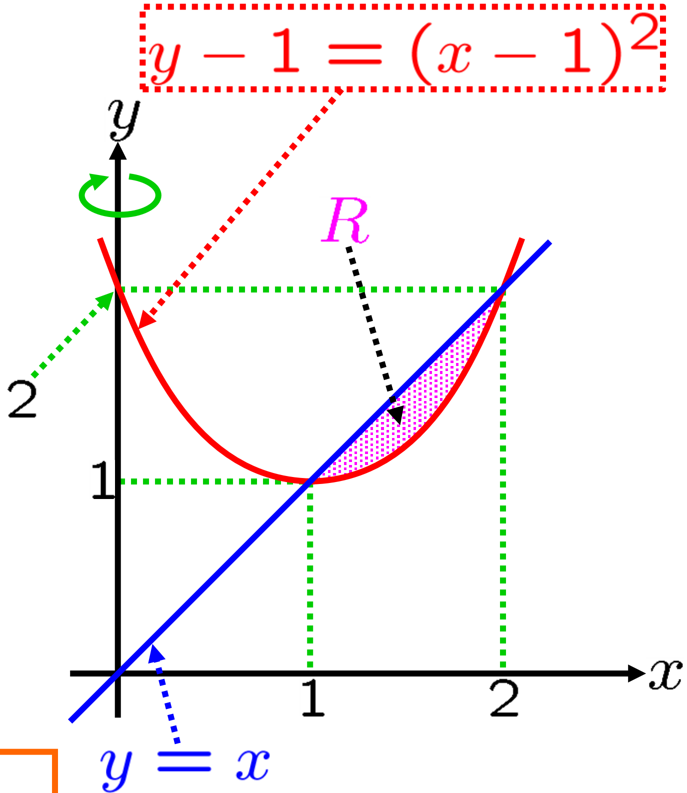
ANSWER:

c.



$$\int_1^2 \pi\left(1+\sqrt{y-1}\right)^2-\pi\left(y^2\right)\,dy$$

$$=\pi\left[-\frac{y^3}{3}+\frac{y^2}{2}+\frac{2(y-1)^{3/2}}{3/2}\right]_{y:\to 1}^{y:\to 2}$$

$$=\pi\left[-\frac{2^3-1^3}{3}+\frac{2^2-1^2}{2}+\frac{2[(1)^{3/2}-0]}{3/2}\right]$$

0720-2. OLD Let R be the region bounded by

$$y-1=(x-1)^2 \text{ and } y=x.$$

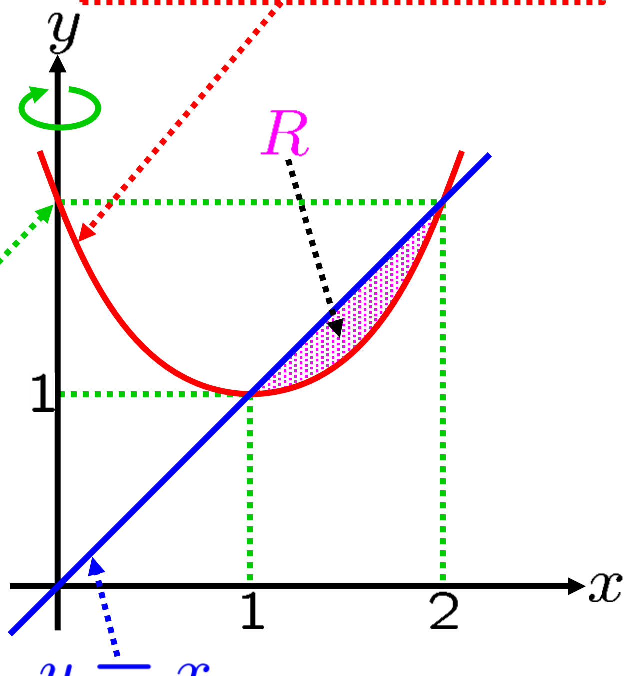
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

ANSWER:

c.



$$\int_1^2 \pi\left(1+\sqrt{y-1}\right)^2-\pi\left(y^2\right)\,dy$$

$$=\pi\left[-\frac{2^3-1^3}{3}+\frac{2^2-1^2}{2}+\frac{2[(1)^{3/2}-0]}{3/2}\right]$$

$$=\pi\left[-\frac{7}{3}+\frac{3}{2}+\frac{2}{3/2}\right]$$

$$=\pi\left[-\frac{7}{3}+\frac{3}{2}+\frac{4}{3}\right]$$

0720-2. OLD Let R be the region bounded by

$$y - 1 = (x-1)^2 \text{ and } y = x.$$

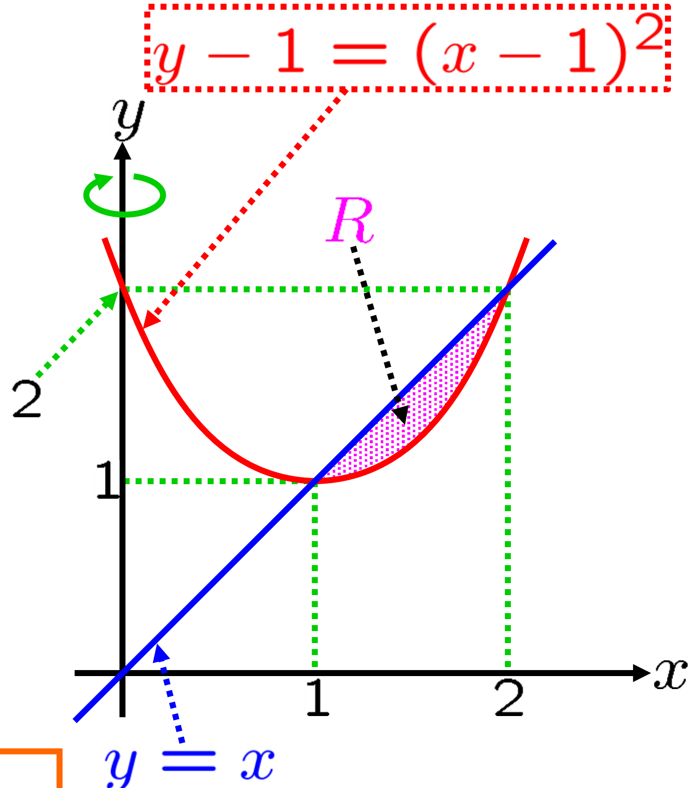
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

c. Find the volume of the solid obtained by rotating R about the y-axis.

ANSWER:

c.



$$\int_1^2 \pi \left(1 + \sqrt{y-1}\right)^2 - \pi\left(y^2\right)\, dy$$

$$= \pi\left[-\frac{7}{3} + \frac{3}{2} + \frac{4}{3}\right]$$

$$= \pi\left[-1 + \frac{3}{2}\right] \quad = \quad \frac{\pi}{2}$$

0720-3. Let R be the region bounded by

OLD

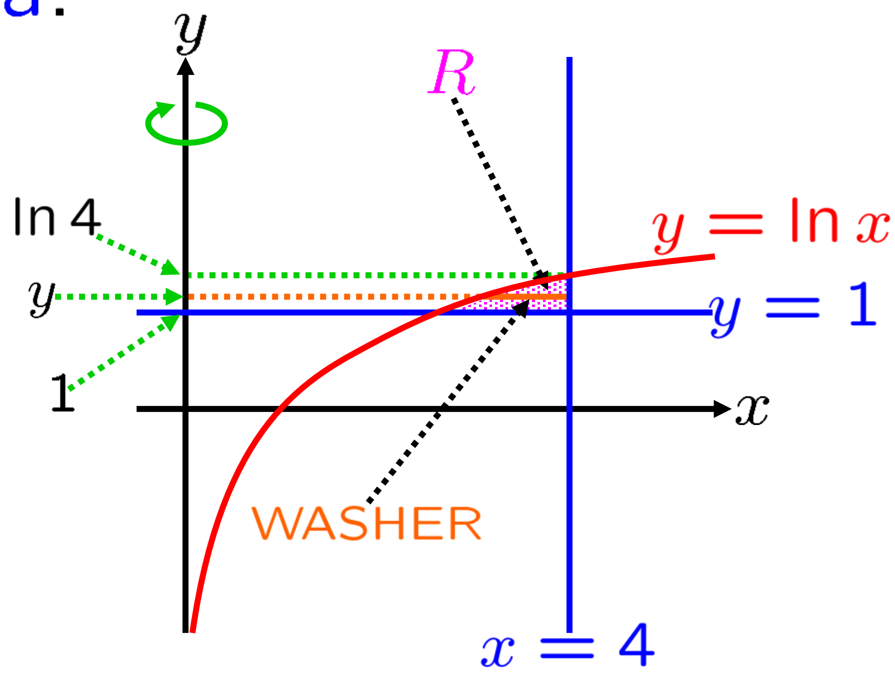
$$y = \ln x,\ x = 4 \text{ and } y = 1.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the y-axis.

ANSWER:

a.



b.

$$\int_1^{\ln 4} \pi(4^2) - \pi(e^y)^2\, dy$$

$$= \pi \int_1^{\ln 4} 16 - e^{2y}\, dy$$

$$= \pi \left[16y - \frac{e^{2y}}{2} \right]_{y:\to 1}^{y:\to \ln 4}$$

$$= \pi \left[\left[16 \ln 4 - \frac{(e^{\ln 4})^2}{2} \right] - \left[16 - \frac{e^2}{2} \right] \right]$$

$$= \pi \left[16 \ln 4 - \frac{4^2}{2} - 16 + \frac{e^2}{2} \right]$$

$$= \pi \left[16 \ln 4 + \frac{e^2}{2} - 24 \right]$$ ■

0720-4. OLD Let R be the region bounded by $y = \sin x$ and $y = 0$ in $0 \le x \le \pi$.

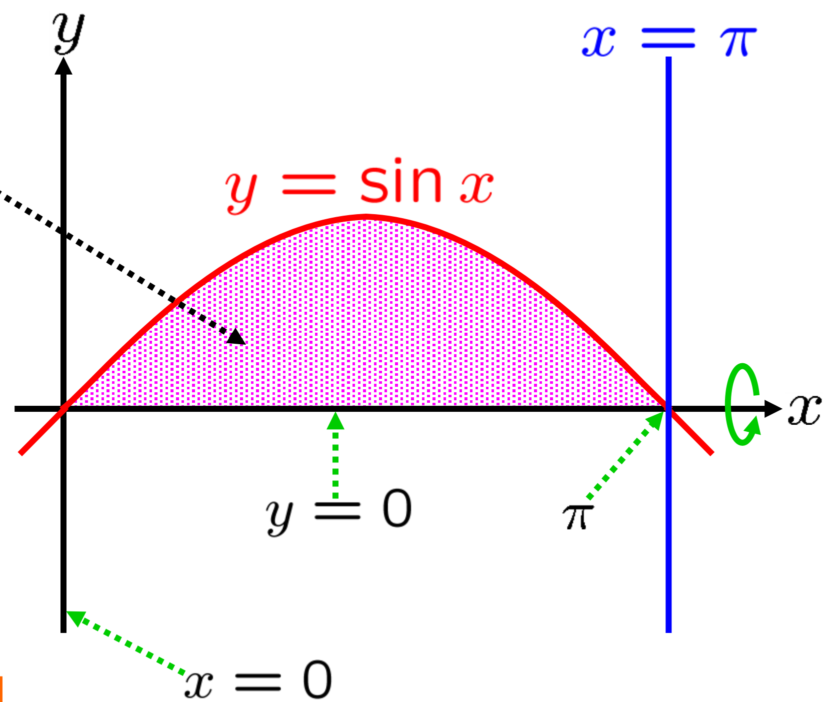
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

Hint: $\sin^2 x = \frac{1 - [\cos(2x)]}{2}$

ANSWER:

a.



b.

$$\int_0^\pi \pi(\sin x)^2\,dx$$

$$= \pi \int_0^\pi \sin^2 x\,dx$$

$$= \pi \int_0^\pi \frac{1 - [\cos(2x)]}{2}\,dx$$

$$= \pi \left[\frac{x - [(\sin(2x))/2]}{2}\right]_{x:\to 0}^{x:\to\pi}$$

$$= \pi \left[\frac{\pi - [0/2]}{2} - \frac{0 - [0/2]}{2}\right]$$

$$= \frac{\pi^2}{2} \blacksquare$$

0720-5. OLD Let R be the region bounded by

$$x^2 + (y-3)^2 = 1.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

Note: This solid is called a torus. It is in the shape of a doughnut.

Hint: Remember that $2\int_{-1}^{1}\sqrt{1-x^2}\,dx$ is known; it is the area enclosed in a circle of radius 1.

0720-5. Let R be the region bounded by
OLD

$$x^2 + (y-3)^2 = 1.$$

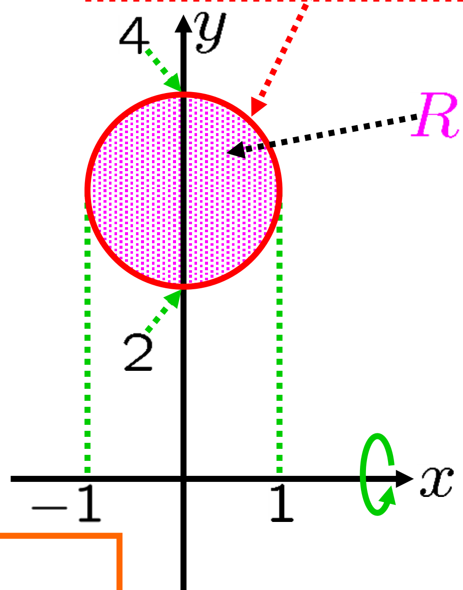
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

Hint: Remember that $2\int_{-1}^{1} \sqrt{1-x^2}\,dx$ is known; it is the area enclosed in a circle of radius 1.

ANSWER:

a. $x^2 + (y-3)^2 = 1$



b. $$\int_{-1}^{1} \pi\left(3+\sqrt{1-x^2}\right)^2 - \pi\left(3-\sqrt{1-x^2}\right)^2\,dx$$

$$= \pi \int_{-1}^{1} \left(9 + 6\sqrt{1-x^2} + 1 - x^2\right) - \left(9 - 6\sqrt{1-x^2} + 1 - x^2\right)\,dx$$

$$= \pi \int_{-1}^{1} 12\sqrt{1-x^2}\,dx$$

0720-5. Let R be the region bounded by
OLD

$$x^2 + (y-3)^2 = 1.$$

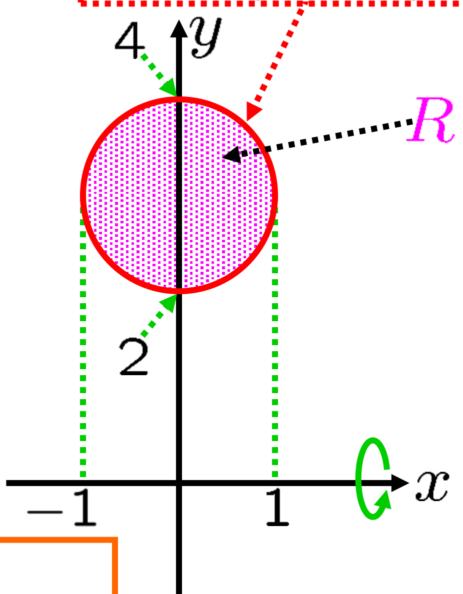
a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the x-axis.

Hint: Remember that $2\int_{-1}^{1}\sqrt{1-x^2}\,dx$ is known; it is the area enclosed in a circle of radius 1.

ANSWER:

a. $x^2 + (y-3)^2 = 1$



b. $\int_{-1}^{1}\pi\left(3+\sqrt{1-x^2}\right)^2 - \pi\left(3-\sqrt{1-x^2}\right)^2\,dx$

$$= \pi\int_{-1}^{1} 12\sqrt{1-x^2}\,dx$$

$$= 6\pi\left[2\int_{-1}^{1}\sqrt{1-x^2}\,dx\right]$$

$$= 6\pi\left[\pi\cdot 1^2\right] \quad = \quad 6\pi^2 \quad ■$$

0720-6. Let R be the region bounded by
OLD

$$y = x^3 \text{ and } x = y^4.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the line $y = -1/2$.

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

0720-6. Let R be the region bounded by

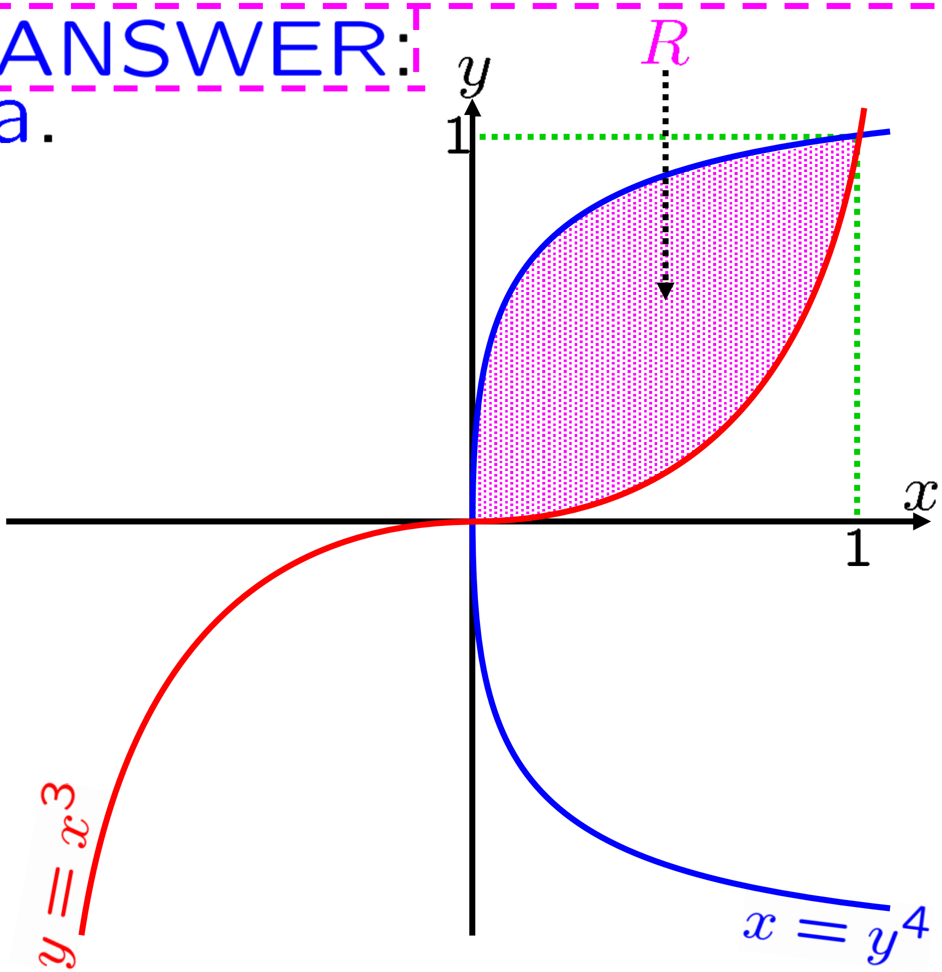
OLD

$$y = x^3 \text{ and } x = y^4.$$

a. Sketch R.

ANSWER:

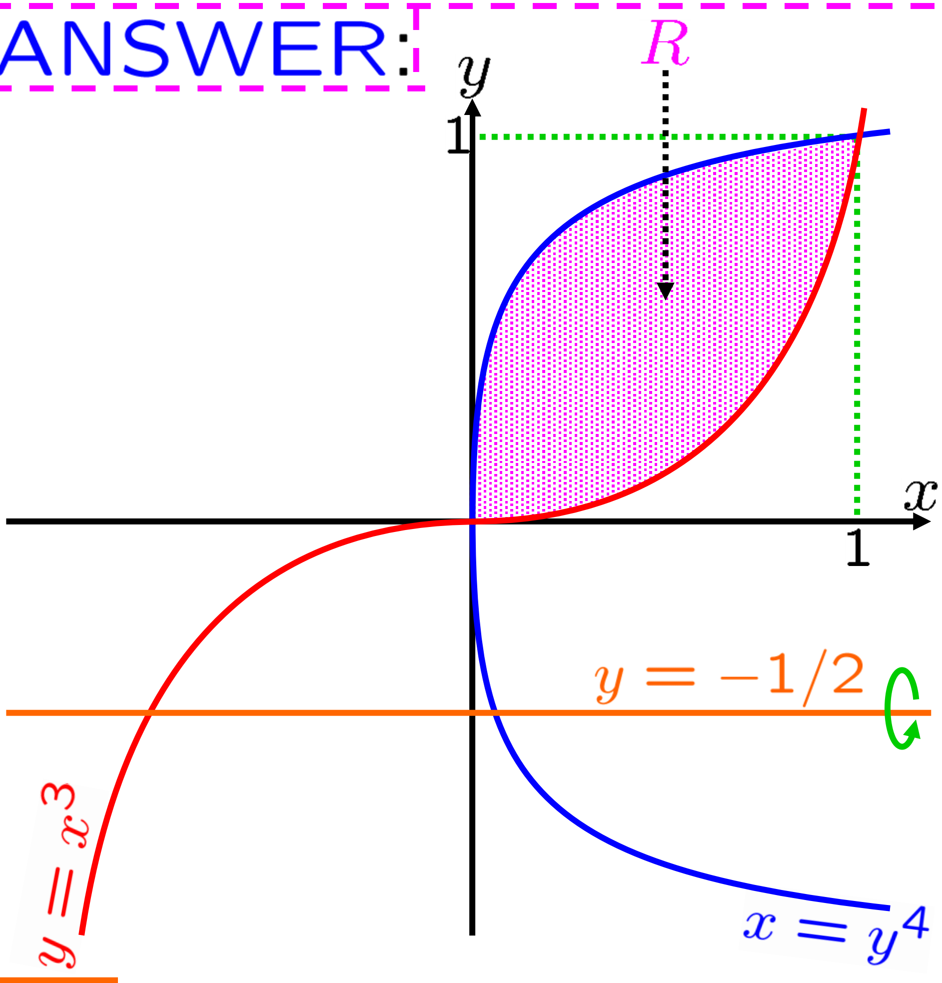
a.



0720-6. OLD Let R be the region bounded by
$y = x^3$ and $x = y^4$.

b. Find the volume of the solid obtained by rotating R about the line $y = -1/2$.

ANSWER:



b.

$$\int_0^1 \pi \left(x^{1/4} + \frac{1}{2}\right)^2 - \pi \left(x^3 + \frac{1}{2}\right)^2 \, dx$$

$$= \pi \int_0^1 \left(x^{1/2} + x^{1/4} + \frac{1}{4}\right) - \left(x^6 + x^3 + \frac{1}{4}\right) dx$$

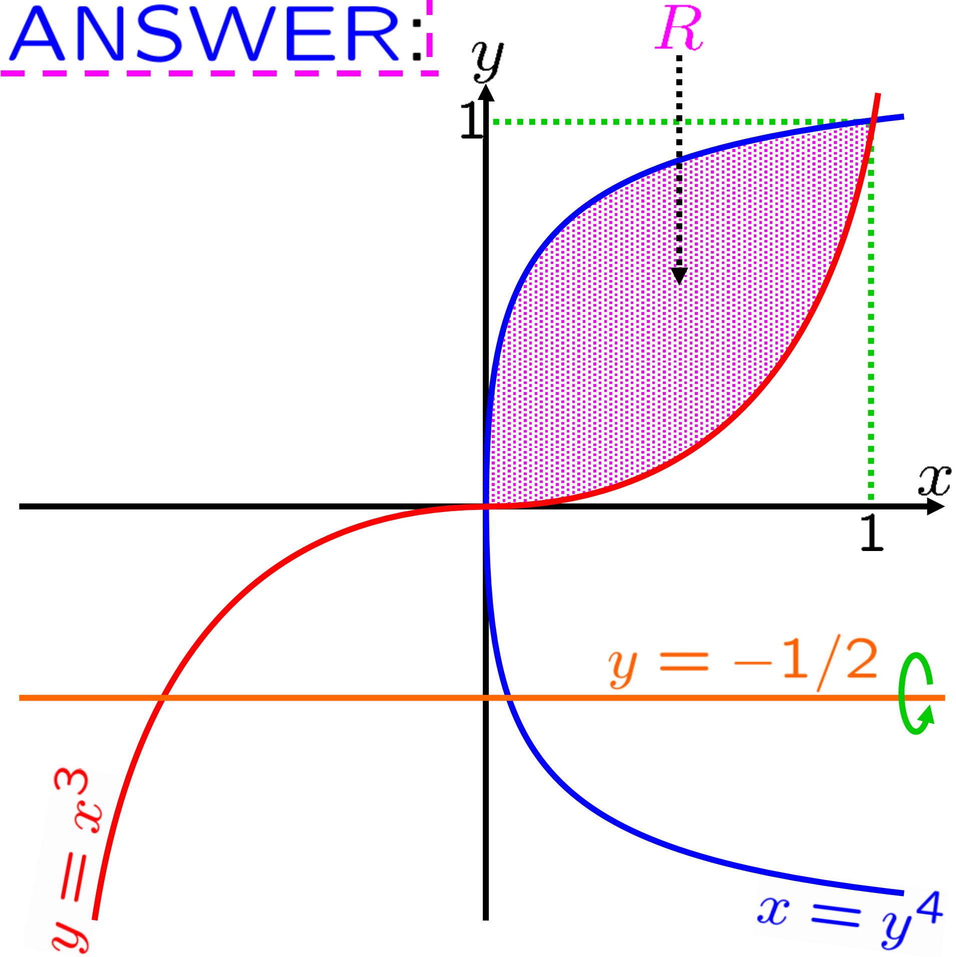
$$= \pi \int_0^1 -x^6 - x^3 + x^{1/2} + x^{1/4} \, dx$$

$$= \pi \left[-\frac{x^7}{7} - \frac{x^4}{4} + \frac{x^{3/2}}{3/2} + \frac{x^{5/4}}{5/4} \right]_{x:\to 0}^{x:\to 1}$$

0720-6. OLD Let R be the region bounded by
$y = x^3$ and $x = y^4$.

b. Find the volume of the solid obtained by rotating R about the line $y = -1/2$.

ANSWER:



b.

$$\int_0^1 \pi\left(x^{1/4}+\frac{1}{2}\right)^2 - \pi\left(x^3+\frac{1}{2}\right)^2 \, dx$$

$$= \pi\left[-\frac{x^7}{7} - \frac{x^4}{4} + \frac{x^{3/2}}{3/2} + \frac{x^{5/4}}{5/4}\right]_{x:\to 0}^{x:\to 1}$$

$$= \pi\left[\left[-\frac{1}{7} - \frac{1}{4} + \frac{1}{3/2} + \frac{1}{5/4}\right] - [0]\right]$$

$$= \pi\left[-\frac{1}{7} - \frac{1}{4} + \frac{2}{3} + \frac{4}{5}\right]$$

$$= \pi\left[-\frac{60}{420} - \frac{105}{420} + \frac{280}{420} + \frac{336}{420}\right]$$

$$= \frac{451\pi}{420}$$

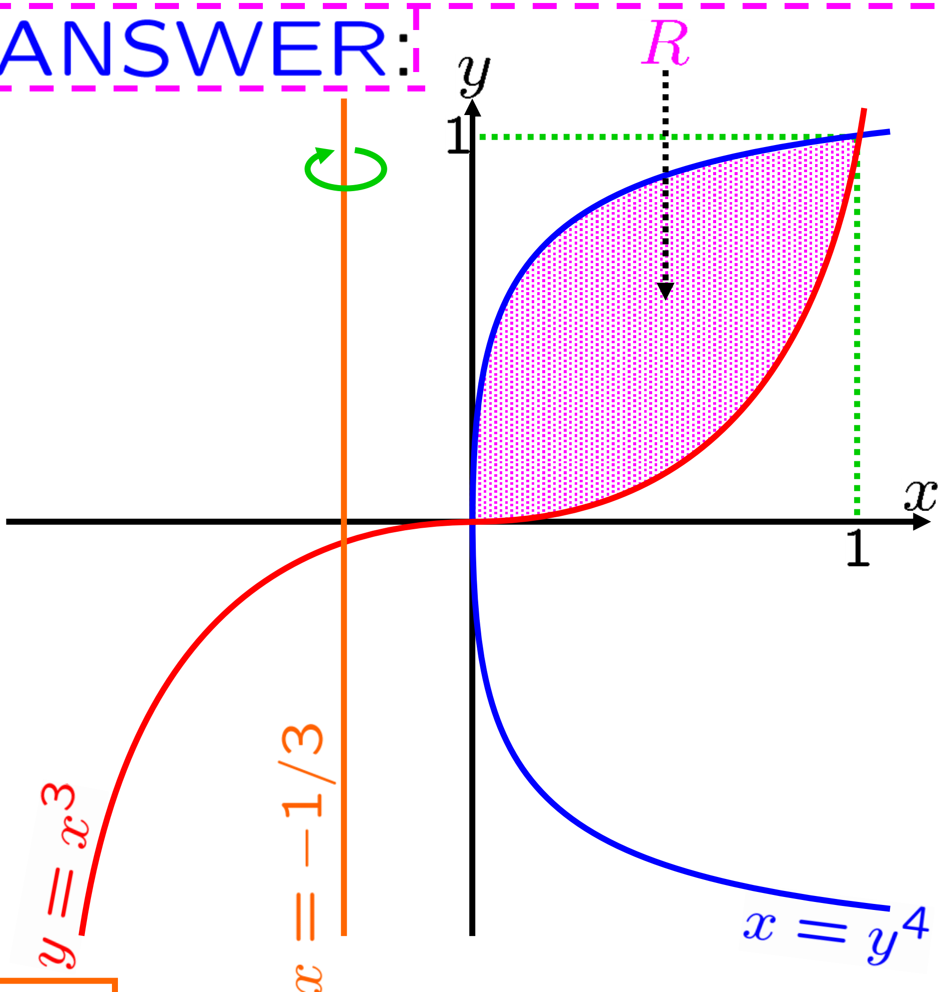
0720-6. Let R be the region bounded by

OLD

$$y = x^3 \text{ and } x = y^4.$$

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

ANSWER:



c.

$$\int_0^1 \pi\left(y^{1/3} + \frac{1}{3}\right)^2 - \pi\left(y^4 + \frac{1}{3}\right)^2 dy$$

$$= \pi \int_0^1 \left(y^{2/3} + \frac{2y^{1/3}}{3} + \frac{1}{9}\right) - \left(y^8 + \frac{2y^4}{3} + \frac{1}{9}\right) dy$$

$$= \pi \int_0^1 -y^8 - \frac{2y^4}{3} + y^{2/3} + \frac{2y^{1/3}}{3}\, dy$$

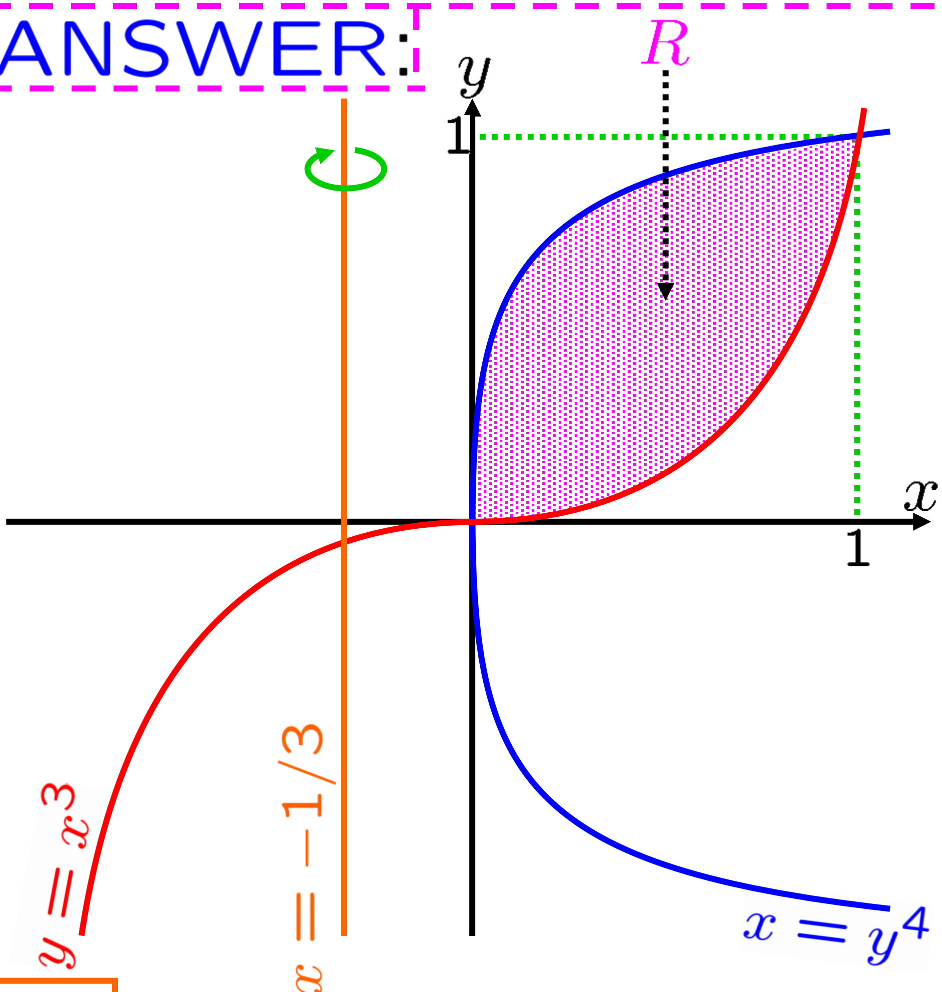
$$= \pi\left[-\frac{y^9}{9} - \frac{2y^5}{15} + \frac{y^{5/3}}{5/3} + \frac{2y^{4/3}}{12/3}\right]_{y:\to 0}^{y:\to 1}$$

$$= \pi\left[\left[-\frac{1}{9} - \frac{2}{15} + \frac{1}{5/3} + \frac{2}{4}\right] - [0]\right]$$

0720-6. OLD Let R be the region bounded by
$$y = x^3 \text{ and } x = y^4.$$

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

ANSWER:



c.
$$\int_0^1 \pi\left(y^{1/3} + \frac{1}{3}\right)^2 - \pi\left(y^4 + \frac{1}{3}\right)^2 dy$$

$$= \pi\left[\left[-\frac{1}{9} - \frac{2}{15} + \frac{1}{5/3} + \frac{2}{4}\right] - [0]\right]$$

$$= \pi\left[-\frac{1}{9} - \frac{2}{15} + \frac{3}{5} + \frac{1}{2}\right]$$

$$= \pi\left[-\frac{10}{90} - \frac{12}{90} + \frac{54}{90} + \frac{45}{90}\right]$$

$$= \frac{77\pi}{90} \blacksquare$$

0720-7. Let R be the region bounded by
OLD

$$y = x^2 \text{ and } x = y^6.$$

a. Sketch R.

b. Find the volume of the solid obtained by rotating R about the line $y = -1/2$.

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

0720-7. Let R be the region bounded by

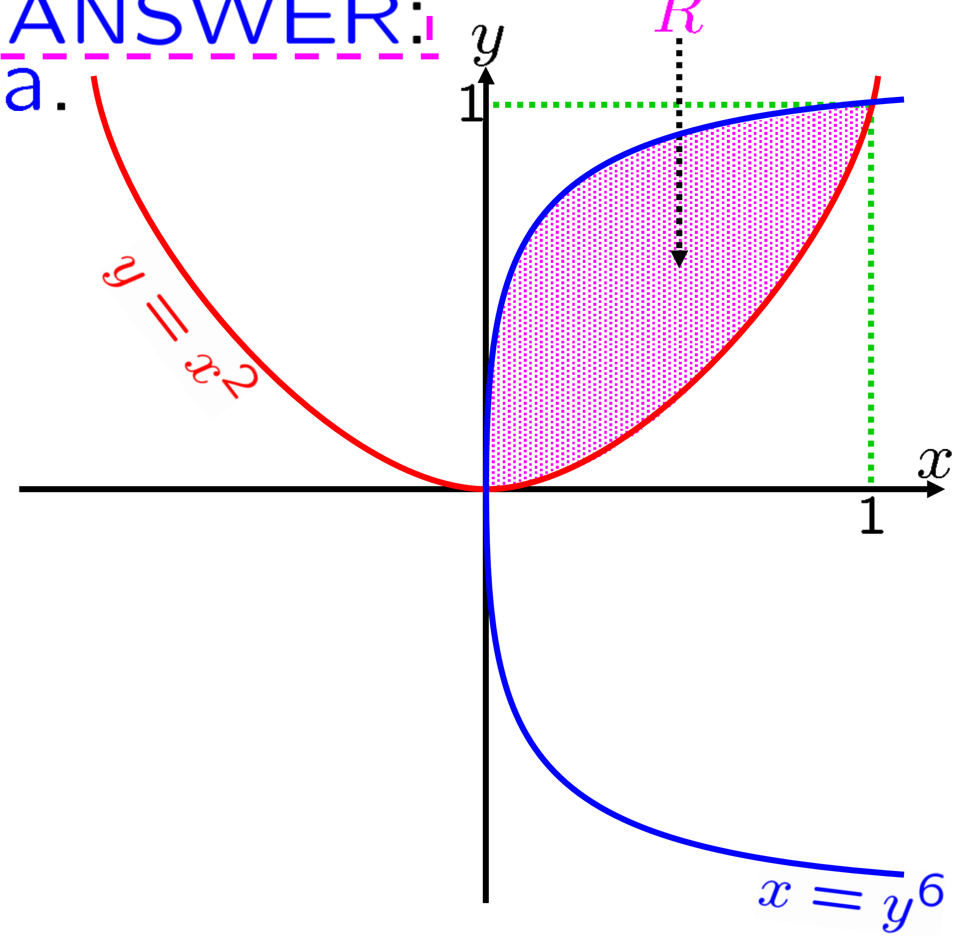
OLD

$$y = x^2 \text{ and } x = y^6.$$

a. Sketch R.

ANSWER:

a.

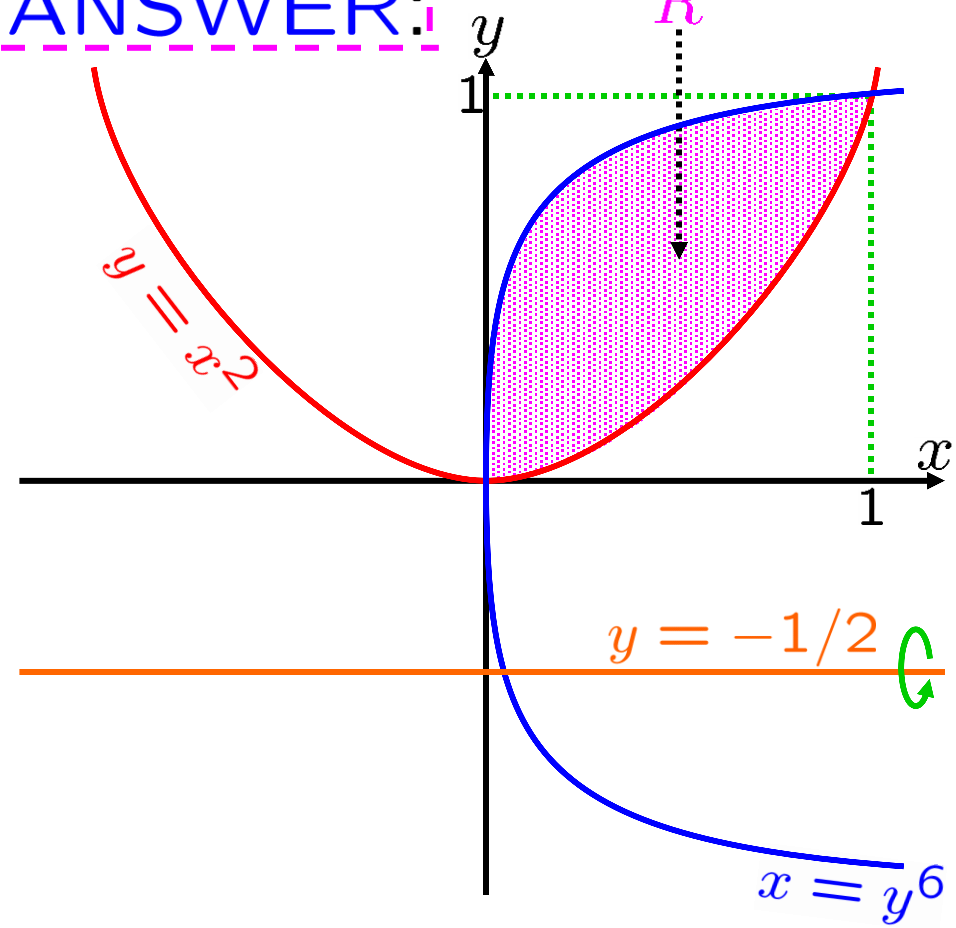


0720-7. Let R be the region bounded by
$y = x^2$ and $x = y^6$.

OLD

b. Find the volume of the solid obtained by rotating R about the line $y = -1/2$.

ANSWER:



b.

$$\int_0^1 \pi \left(x^{1/6} + \frac{1}{2}\right)^2 - \pi \left(x^2 + \frac{1}{2}\right)^2 \, dx$$

$$= \pi \int_0^1 \left(x^{1/3} + x^{1/6} + \frac{1}{4}\right) - \left(x^4 + x^2 + \frac{1}{4}\right) dx$$

$$= \pi \int_0^1 -x^4 - x^2 + x^{1/3} + x^{1/6} \, dx$$

$$= \pi \left[-\frac{x^5}{5} - \frac{x^3}{3} + \frac{x^{4/3}}{4/3} + \frac{x^{7/6}}{7/6} \right]_{x:\to 0}^{x:\to 1}$$

0720-7. Let R be the region bounded by

OLD

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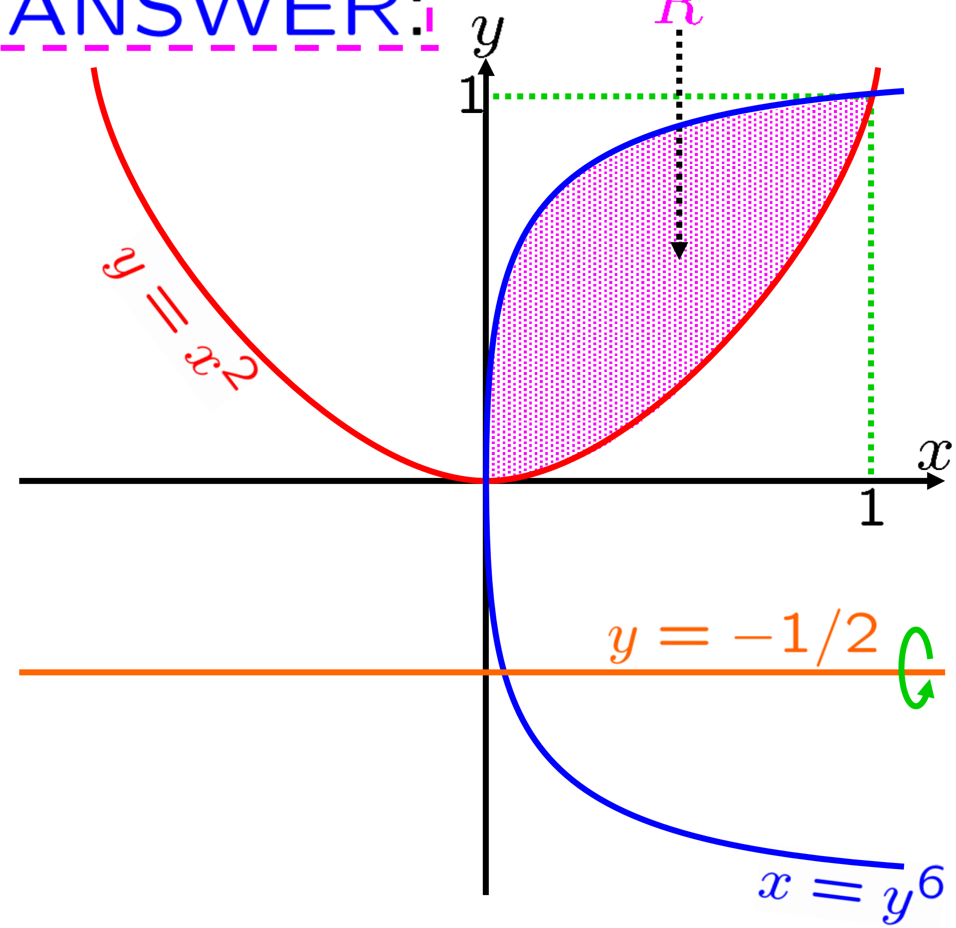


b.

$$\int_0^1 \pi\left(x^{1/6}+\frac{1}{2}\right)^2 - \pi\left(x^2+\frac{1}{2}\right)^2\,dx$$

$$= \pi\left[-\frac{x^5}{5}-\frac{x^3}{3}+\frac{x^{4/3}}{4/3}+\frac{x^{7/6}}{7/6}\right]_{x:\to 0}^{x:\to 1}$$

$$= \pi\left[\left[-\frac{1}{5}-\frac{1}{3}+\frac{1}{4/3}+\frac{1}{7/6}\right]-[0]\right]$$

$$= \pi\left[-\frac{1}{5}-\frac{1}{3}+\frac{3}{4}+\frac{6}{7}\right]$$

$$= \pi\left[-\frac{84}{420}-\frac{140}{420}+\frac{315}{420}+\frac{360}{420}\right]$$

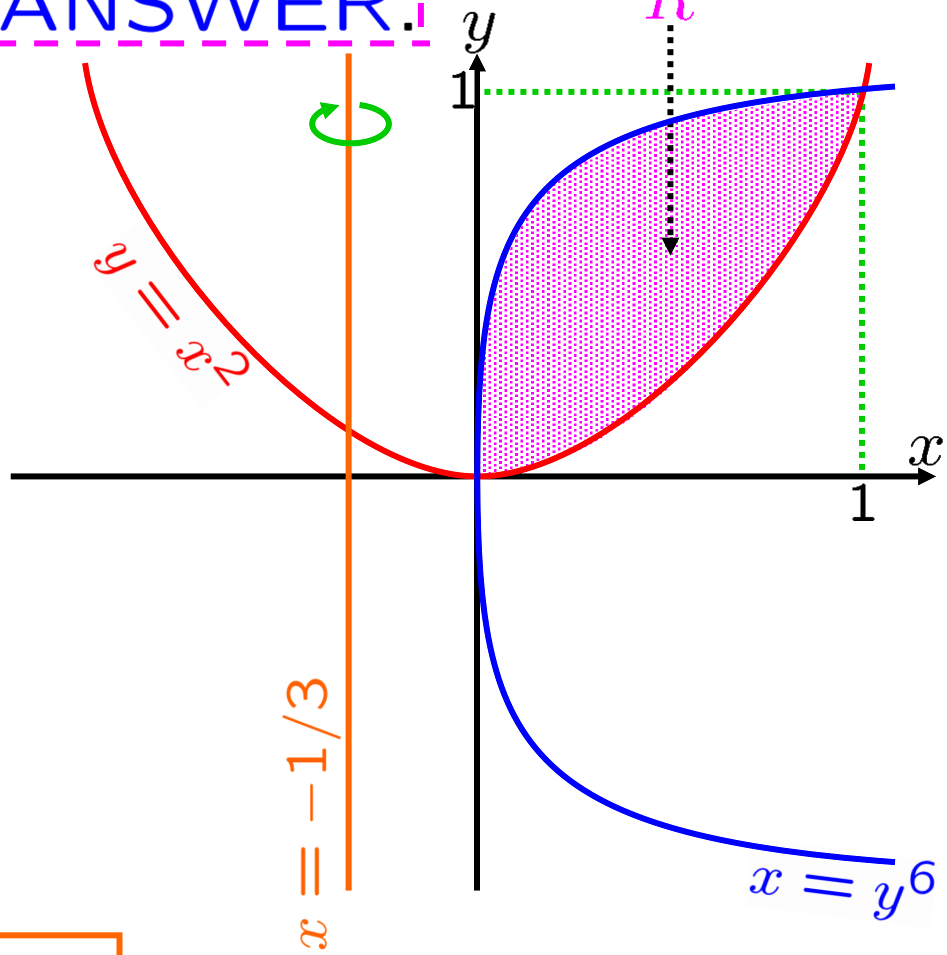
$$= \frac{451\pi}{420}$$

0720-7. Let R be the region bounded by
$y = x^2$ and $x = y^6$.

OLD

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

ANSWER:



c.

$$\int_0^1 \pi\left(y^{1/2}+\frac{1}{3}\right)^2-\pi\left(y^6+\frac{1}{3}\right)^2\,dy$$

$$=\pi\int_0^1\left(y+\frac{2y^{1/2}}{3}+\frac{1}{9}\right)-\left(y^{12}+\frac{2y^6}{3}+\frac{1}{9}\right)\,dy$$

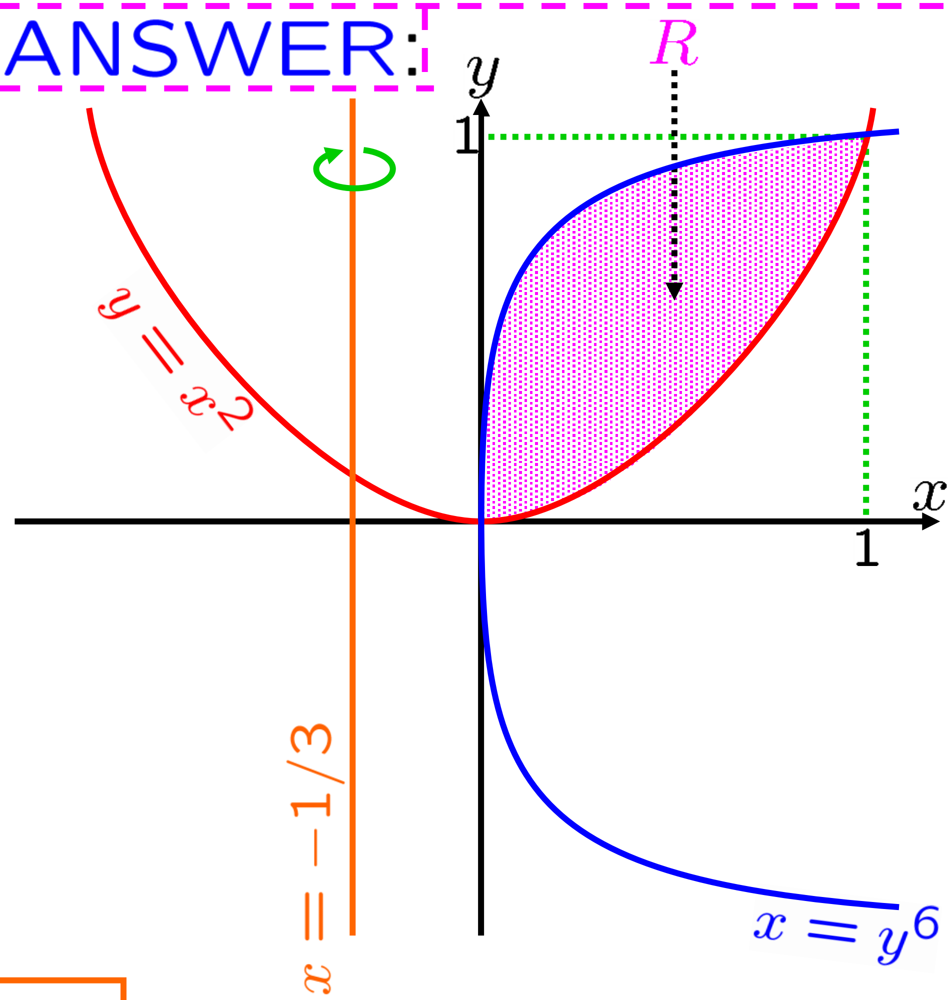
$$=\pi\int_0^1 -y^{12}-\frac{2y^6}{3}+y+\frac{2y^{1/2}}{3}\,dy$$

$$=\pi\left[-\frac{y^{13}}{13}-\frac{2y^7}{21}+\frac{y^2}{2}+\frac{2y^{3/2}}{9/2}\right]_{y:\to 0}^{y:\to 1}$$

0720-7. OLD. Let R be the region bounded by $y = x^2$ and $x = y^6$.

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

ANSWER:



c.

$$\int_0^1 \pi\left(y^{1/2}+\frac{1}{3}\right)^2 - \pi\left(y^6+\frac{1}{3}\right)^2\,dy$$

$$= \pi\left[-\frac{y^{13}}{13}-\frac{2y^7}{21}+\frac{y^2}{2}+\frac{2y^{3/2}}{9/2}\right]_{y:\to 0}^{y:\to 1}$$

$$= \pi\left[\left[-\frac{1}{13}-\frac{2}{21}+\frac{1}{2}+\frac{2}{9/2}\right]-[0]\right]$$

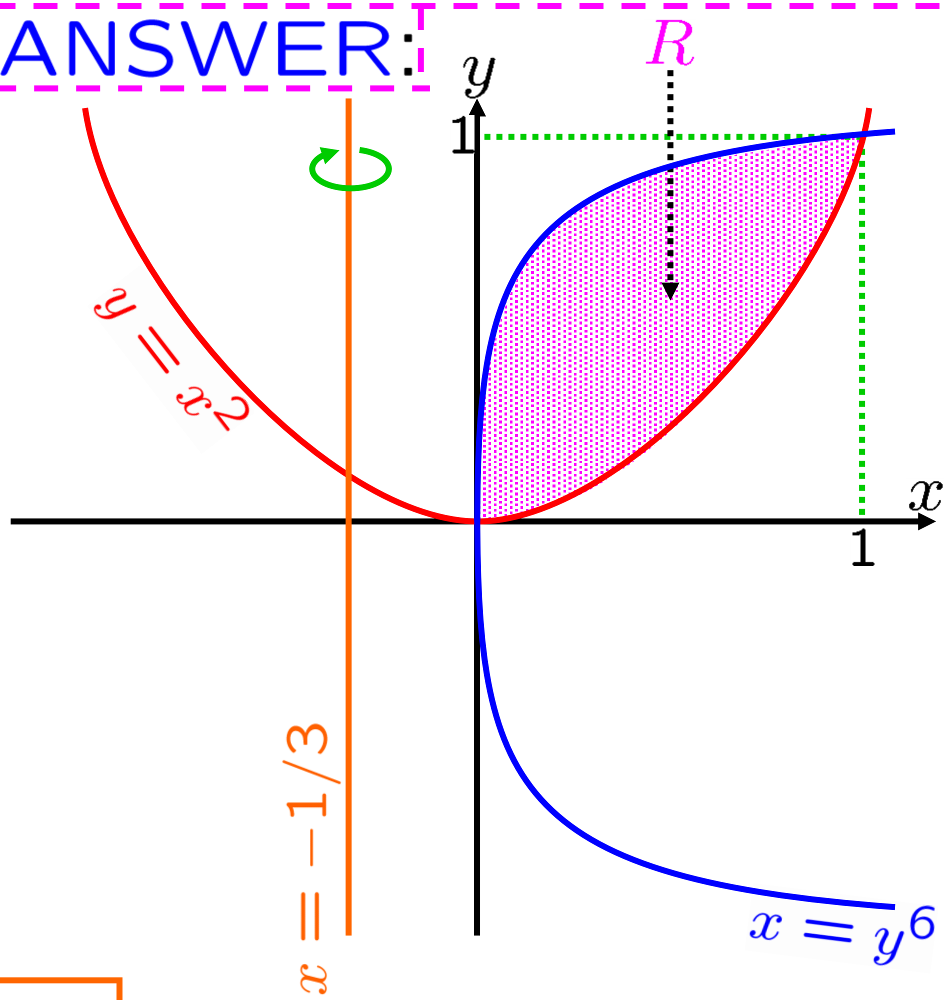
$$= \pi\left[-\frac{1}{13}-\frac{2}{21}+\frac{1}{2}+\frac{4}{9}\right]$$

0720-7. Let R be the region bounded by
OLD

$y = x^2$ and $x = y^6$.

c. Find the volume of the solid obtained by rotating R about the line $x = -1/3$.

ANSWER:



c.

$$\int_0^1 \pi \left(y^{1/2} + \frac{1}{3}\right)^2 - \pi \left(y^6 + \frac{1}{3}\right)^2 \, dy$$

$$= \pi\left[-\frac{1}{13} - \frac{2}{21} + \frac{1}{2} + \frac{4}{9}\right]$$

$$= \pi\left[-\frac{126}{1638} - \frac{156}{1638} + \frac{819}{1638} + \frac{728}{1638}\right]$$

$$= \frac{1265\pi}{1638}$$

OLD

0720-8. Let R be the region bounded by

$$y = 4\cos x,\ y = e^x \text{ in } 0 \le x \le \pi/4.$$

Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating R about the line $y = 5$.

ANSWER:

$\forall x \in [0, \pi/4]$,

$$5 > 4\cos x \ge 4\cos(\pi/4) > e^{\pi/4} \ge e^x$$

$$\int_0^{\pi/4} \pi\,(5 - e^x)^2 - \pi\,(5 - 4\cos x)^2\, dx$$ ■

OLD

0720-9. Describe the solid of revolution whose volume is given by

$$\pi \int_1^2 \left(9e^{8x} - 4e^{2x}\right)\, dx.$$

Do not evaluate this integral.

ANSWER:

This is the solid obtained by revolving the region bounded by

$$y = 3e^{4x},\ y = 2e^{x} \text{ in } 1 \le x \le 2$$

about the x-axis. ■

0720-10. Describe the solid of revolution
OLD
whose volume is given by

$$\pi \int_{\pi/2}^{\pi} (2+\sin x)^2 - 4\,dx.$$

Do not evaluate this integral.

ANSWER:

This is the solid obtained by revolving the region bounded by

$$y = 2+\sin x,\ y = 2 \text{ in } \pi/2 \leq x \leq \pi$$

about the x-axis. ■

ALTERNATE ANSWER:

This is the solid obtained by revolving the region bounded by

$$y = \sin x,\ y = 0 \text{ in } \pi/2 \leq x \leq \pi$$

about the line $y = -2$. ■

0720-11. A solid S sits above a horizontal plane P. $\forall x \geq 0$, let P_x be the horizontal plane that is x units above P. Suppose that S lies between P_1 and P_2. Suppose, also, that $\forall x \in [1, 2]$, the intersection of S and P_x is the region inside an ellipse

OLD

whose major axis has length x

and whose minor axis has length e^{2x^2}.

Compute the volume of S.

Hint: Remember that if a and b are the major and minor axes of an ellipse E, then the area inside E is $\pi ab/4$.

0720-11. A solid S sits above a horizontal
OLD
plane P. $\forall x \geq 0$, let P_x be the horizontal plane that is x units above P. Suppose that S lies between P_1 and P_2. Suppose, also, that $\forall x \in [1, 2]$, the intersection of S and P_x is the region inside an ellipse

whose major axis has length x

and whose minor axis has length e^{2x^2}.

Compute the volume of S.

Hint: Remember that if a and b are the major and minor axes of an ellipse E, then the area inside E is $\pi ab/4$.

ANSWER:

Area of the ellipse in P_x: $\pi x e^{2x^2}/4$

Volume of S: $\int_1^2 \left(\pi x e^{2x^2}/4 \right) \, dx$

0720-11. . . . Compute the volume of S.
OLD

ANSWER: Volume of S: $\int_1^2 \left(\pi x e^{2x^2}/4\right)\,dx$

Let $u = x^2$. Then $du = 2x\,dx$.

$$\int_1^2 \left(\pi x e^{2x^2}/4\right)\,dx = \int_{1^2}^{2^2} \left(\pi e^{2u}/4\right)\frac{du}{2}$$

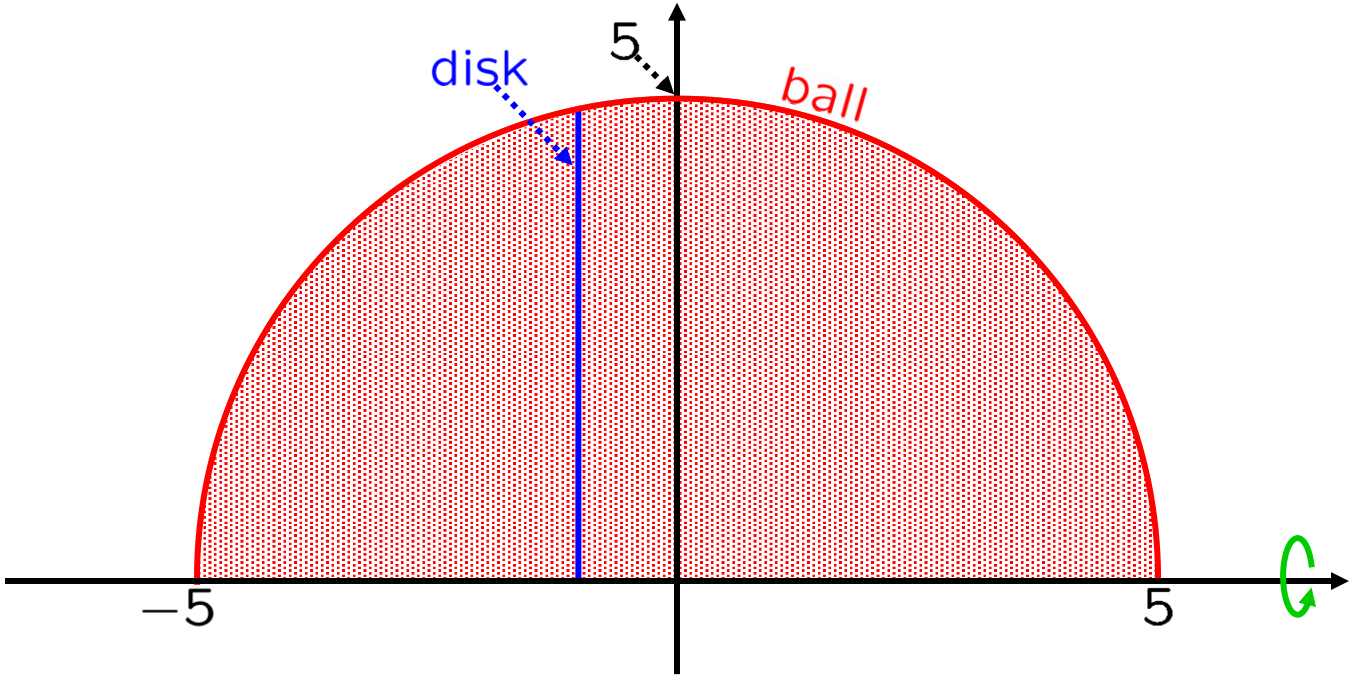
$$= \frac{\pi}{8}\int_1^4 e^{2u}\,du$$

$$= \frac{\pi}{8}\left[\frac{e^{2u}}{2}\right]_{u:\to 1}^{u:\to 4}$$

$$= \frac{\pi}{8}\left[\frac{e^8}{2} - \frac{e^2}{2}\right] \blacksquare$$

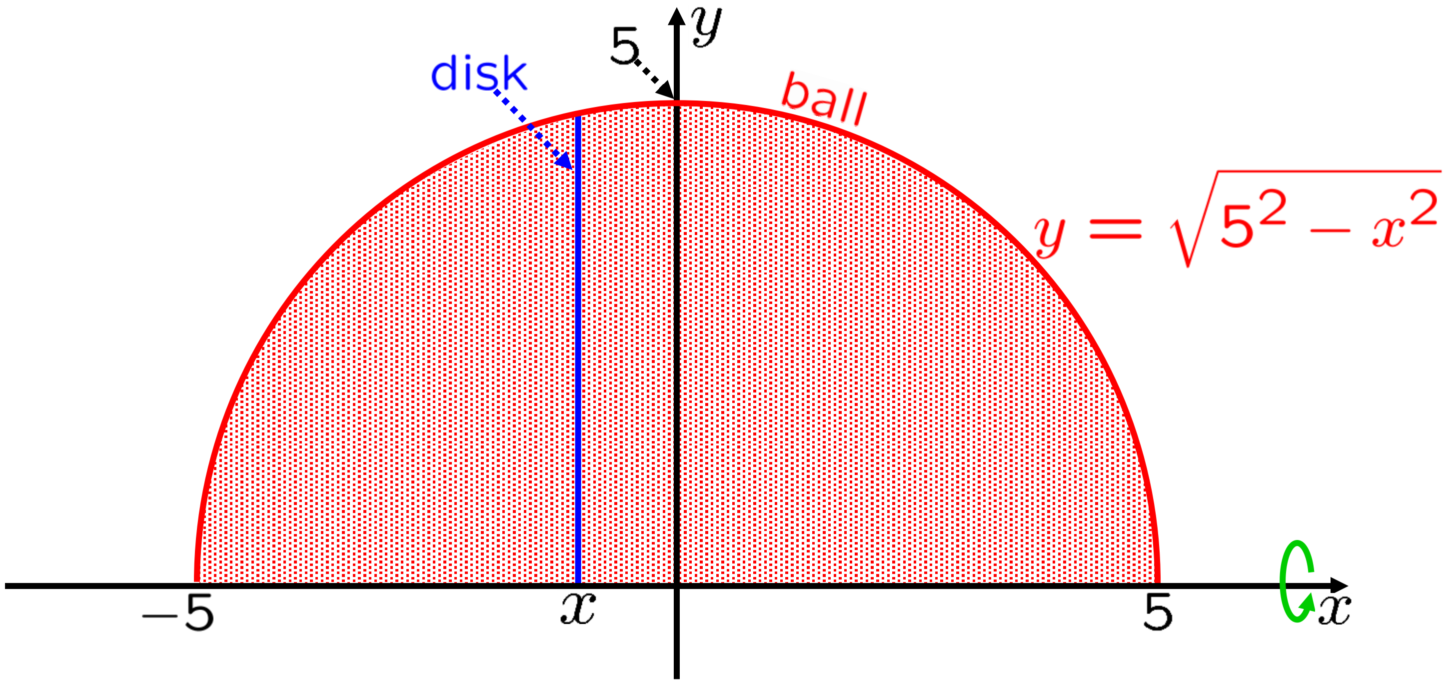
0720-12. Using the disk method, find the volume in a ball of radius 5, following the diagram shown below.

OLD



0720-12. Using the disk method, find the volume in a ball of radius 5, following the diagram shown below.

OLD



ANSWER: $\int_{-5}^{5} \pi \left(\sqrt{5^2 - x^2}\right)^2 \, dx$

0720-12. Using the disk method, find the volume in a ball of radius 5, . . .

OLD

ANSWER:

$$\int_{-5}^{5} \pi \left(\sqrt{25 - x^2}\right)^2 dx = \pi \int_{-5}^{5} 5^2 - x^2 \, dx$$

$5^2 - x^2$ is even in x

$$= 2\pi \int_{0}^{5} 5^2 - x^2 \, dx$$

$$= 2\pi \left[5^2 x - \frac{x^3}{3} \right]_{x:\to 0}^{x:\to 5}$$

$$= 2\pi \left[\left(5^2 \cdot 5 - \frac{5^3}{3} \right) - (0) \right]$$

$$= 2\pi \left[5^3 - \frac{5^3}{3} \right]$$

0720-12. Using the disk method, find the volume in a ball of radius 5, . . .

OLD

ANSWER:

$$\int_{-5}^{5} \pi \left(\sqrt{25 - x^2}\right)^2 \, dx = 2\pi \left[5^3 - \frac{5^3}{3}\right]$$

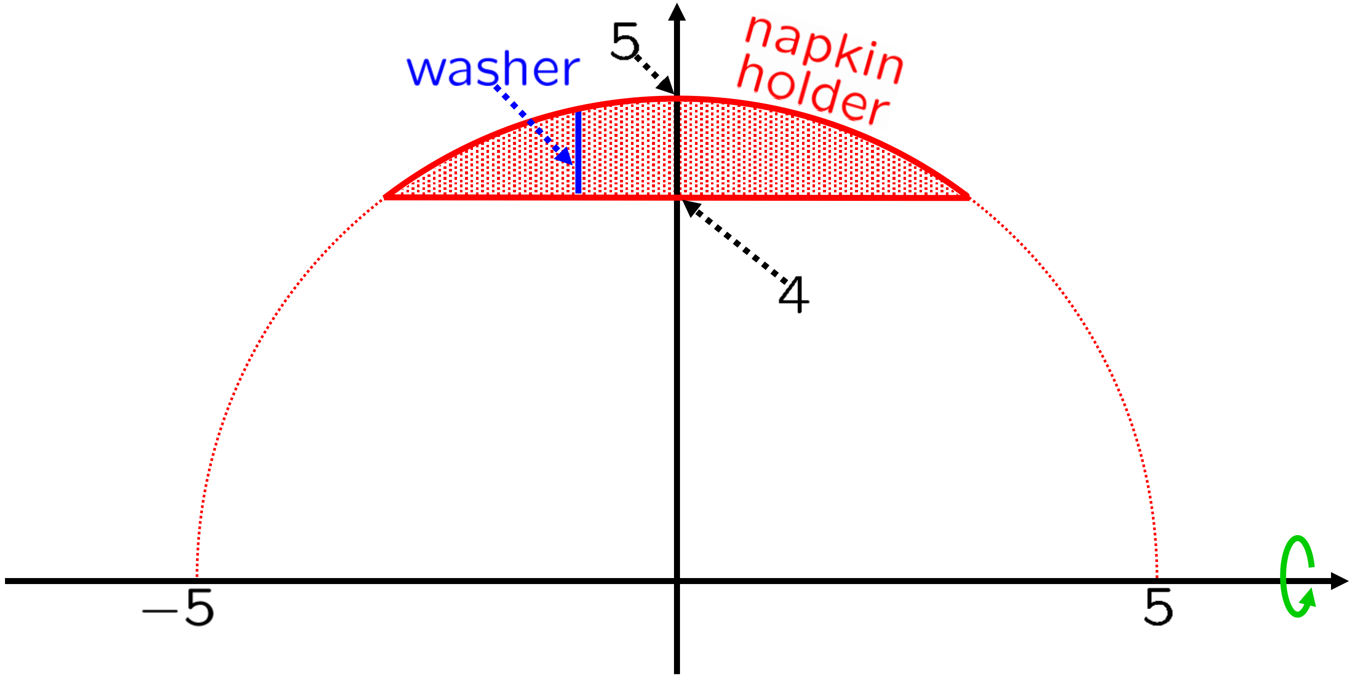
$$= 2\pi \left[1 - \frac{1}{3}\right] \left[5^3\right]$$

$$= 2\pi \left[\frac{2}{3}\right] \left[5^3\right]$$

$$= \frac{4}{3}\pi \left[5^3\right] \blacksquare$$

0720-13. We create a napkin holder by drilling a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the washer method, find its volume.

OLD



0720-13. We create a napkin holder by drilling a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the washer method, find its volume.

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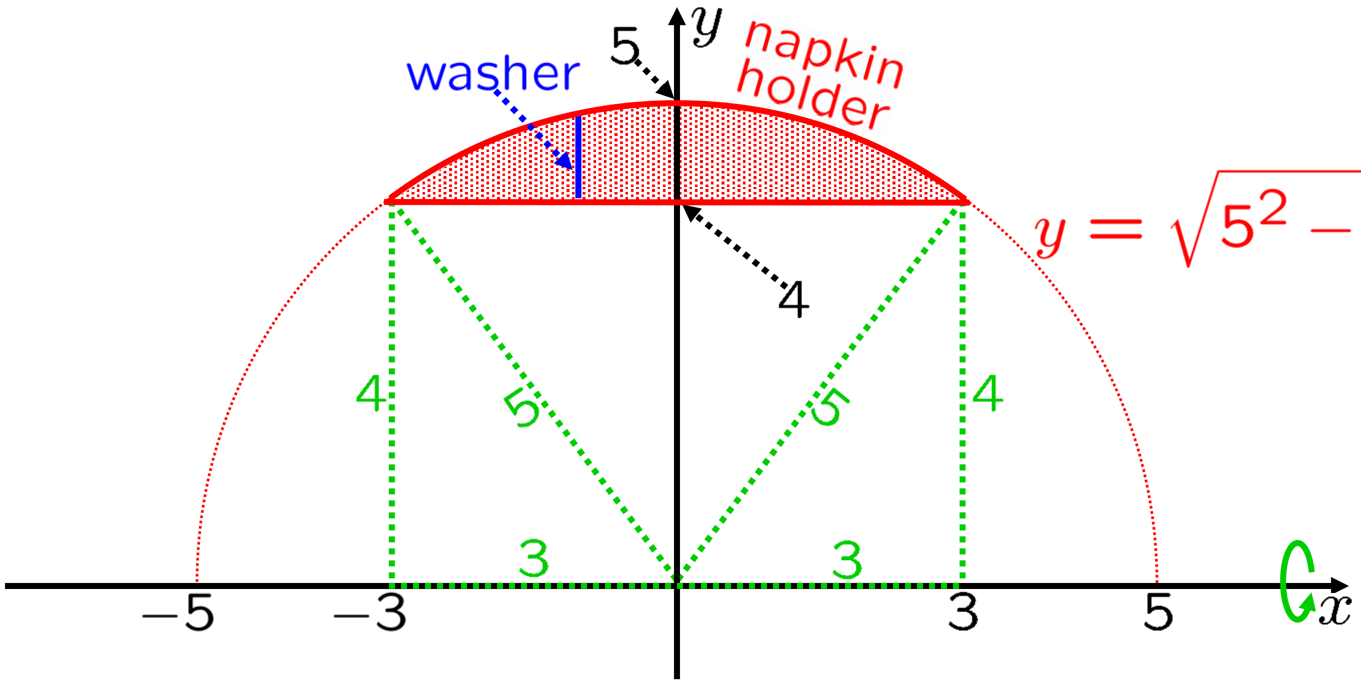

ANSWER: $$\int_{-3}^{3} \pi \left(\sqrt{5^2 - x^2}\right)^2 - \pi \left[4^2\right] \, dx$$

0720-13. We create a napkin holder . . .
OLD Using the washer method, find its volume.

ANSWER: $$\int_{-3}^{3} \pi \left(\sqrt{5^2 - x^2} \right)^2 - \pi \left[4^2 \right] \, dx$$

$$= \pi \int_{-3}^{3} 5^2 - x^2 - 4^2 \, dx$$

$$= \pi \int_{-3}^{3} 3^2 - x^2 \, dx$$

$3^2 - x^2$ is even in x

$$= 2\pi \int_{0}^{3} 3^2 - x^2 \, dx$$

$$= 2\pi \left[3^2 x - \frac{x^3}{3} \right]_{x:\to 0}^{x:\to 3}$$

$$= 2\pi \left[\left(3^2 \cdot 3 - \frac{3^3}{3} \right) - (0) \right]$$

0720-13. We create a napkin holder . . .
OLD Using the washer method, find its volume.

ANSWER: $\int_{-3}^{3} \pi \left(\sqrt{5^2 - x^2}\right)^2 - \pi \left[4^2\right] \, dx$

$$= 2\pi \left[\left(3^2 \cdot 3 - \frac{3^3}{3} \right) - (0) \right]$$

$$= 2\pi \left[3^3 - \frac{3^3}{3} \right]$$

$$= 2\pi \left[1 - \frac{1}{3} \right] \left[3^3 \right]$$

$$= 2\pi \left[\frac{2}{3} \right] \left[3^3 \right]$$

$$= \frac{4}{3}\pi \left[3^3 \right] \blacksquare$$