# CALCULUS

## Standard notation

# Standard Notation

$\forall$ stands for for all
or, sometimes, for any

$\exists$ stands for there exists
or, sometimes, there exist

s.t. stands for such that

$\Rightarrow$ stands for implies

"$A \Rightarrow B$" is equivalent to "if $A$ then $B$".

iff and $\Leftrightarrow$ both stand for if and only if

"$A \Leftrightarrow B$" is equivalent to "both $A \Rightarrow B$ and $B \Rightarrow A$".

Next: basic notation in set theory

# Standard Notation

$\textbf{scalar} := \overbrace{\phantom{xx}}^{\text{real}}\text{number}$

$\boxed{\emptyset} = \{\quad\}$ is the set with no elements

union: $\{4,5,6\} \cup \{5,6,7,8\} = \{4,5,6,7,8\}$

intersection: $\{4,5,6\} \cap \{5,6,7,8\} = \{5,6\}$

complement: $\{4,5,6\} \setminus \{5,6,7,8\} = \{4\}$

$\in$ stands for is an element of

$7 \in \{7,8,9\}$

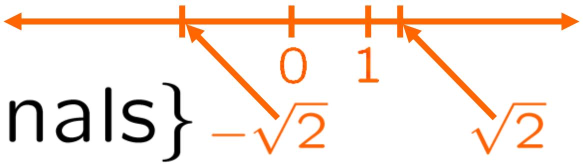
$6 \notin \{7,8,9\}$

is not an elt of

$\boxed{\mathbb{Z}} := \{\text{integers}\} = \{\ldots,-2,-1,0,1,2,\ldots\}$

$\boxed{\mathbb{R}} := \{\text{real numbers}\}$ "="

$= \{\text{rationals}\} \cup \{\text{irrationals}\}$



$\boxed{\mathbb{Q}} := \{\text{rational numbers}\}$

$\boxed{\mathbb{C}} := \{\text{complex numbers}\}$ not used in this class

# Standard Notation

Next: intervals

$A \subseteq B$ means: $\forall x \in A,\ x \in B$.

read: $A$ “is a subset of” $B$

$B \supseteq A$ means: $\forall x \in A,\ x \in B$.

read: $B$ “is a superset of” $A$

$$\boxed{A \subseteq B \quad \Leftrightarrow \quad B \supseteq A}$$

*e.g.*: $\mathbb{Z}\subseteq\mathbb{Q}\subseteq\mathbb{R}\subseteq\mathbb{C}$

$\mathbb{C}\supseteq\mathbb{R}\supseteq\mathbb{Q}\supseteq\mathbb{Z}$

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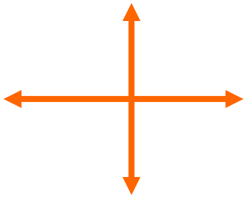


# Standard Notation

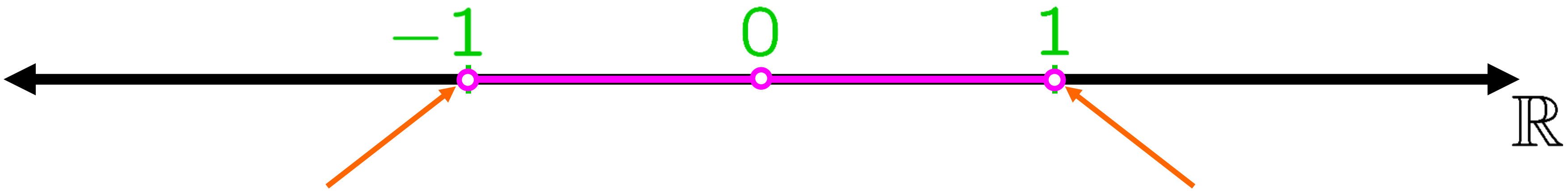
An **interval** is a subset of $\mathbb{R}$ with no “breaks”.

WARNING: This is a set, NOT a point in the plane.

$$(-1,1) = \{x \in \mathbb{R} \,|\, -1 < x < 1\}$$

interval

$(-1,1)\backslash\{0\}$ is not an interval.



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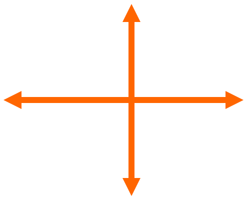

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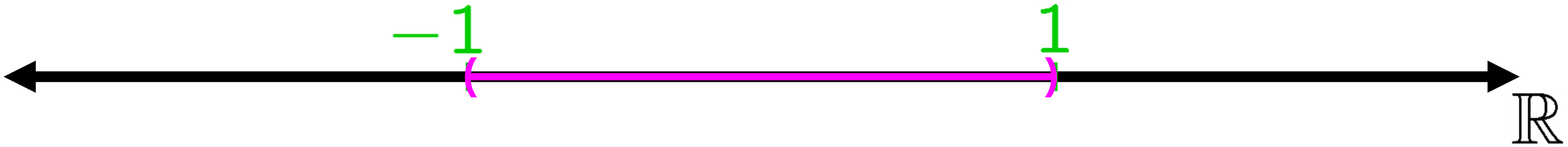

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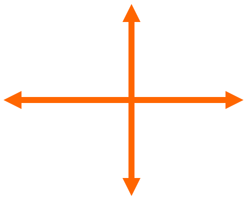

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# Standard Notation

An **open interval** is a set of the form $(a, b)$, where $-\infty \leq a < b \leq \infty$.

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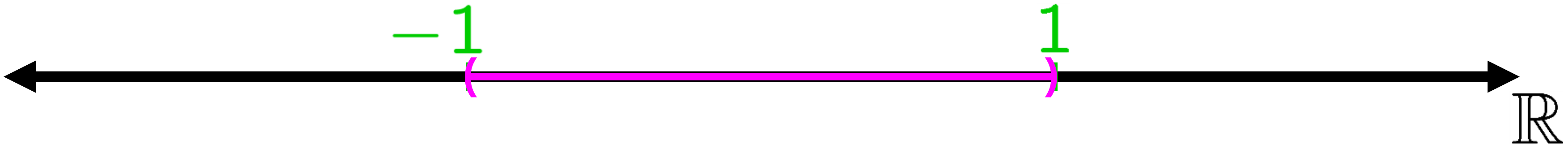
$$(-1, 1) = \{x \in \mathbb{R} \mid -1 < x < 1\}$$

open interval



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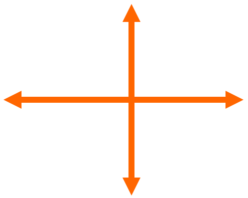

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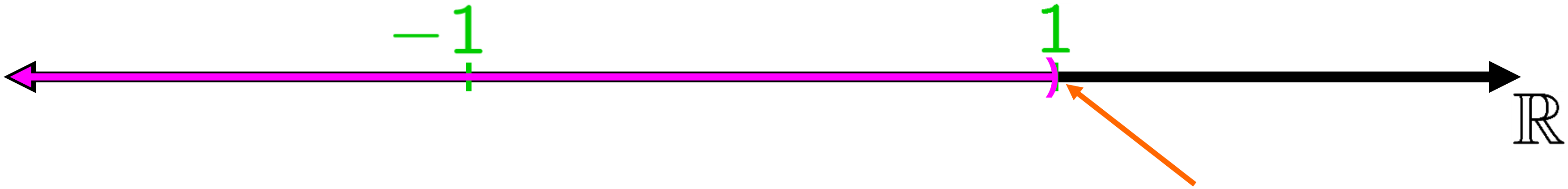
$$(-\infty, 1) = \{x \in \mathbb{R} \,|\, x < 1\}$$

open, unbounded interval



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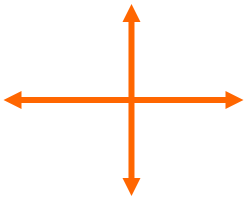

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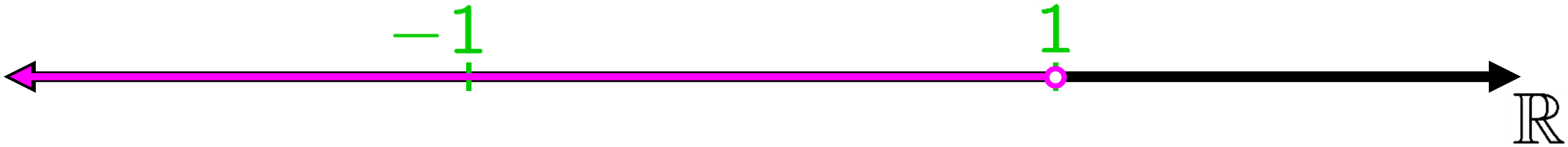

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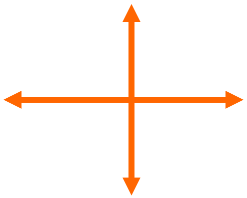

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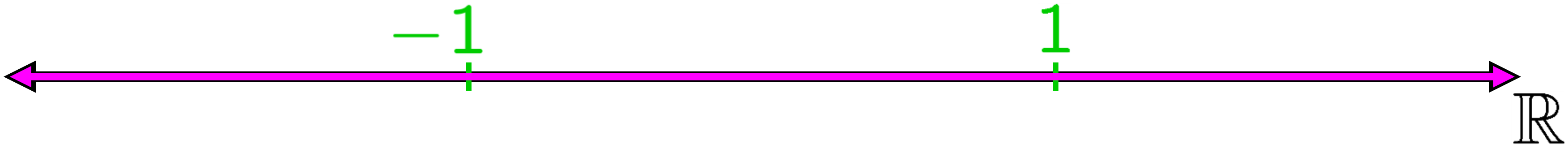
$$(-\infty, \infty) = \mathbb{R}$$

open, unbounded interval



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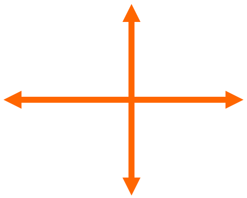
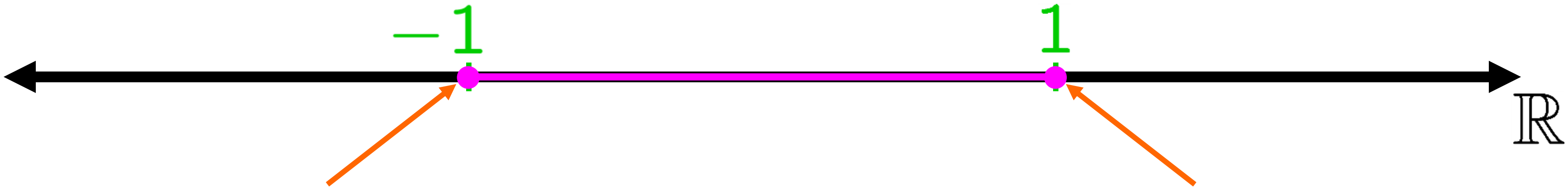


# Standard Notation

A **compact interval** is a set of the form $[a,b]$, where $-\infty < a \leq b < \infty$.

$$[-1,1] = \{x \in \mathbb{R} \,|\, -1 \leq x \leq 1\}$$



$\boxed{\mathbb{Z}} := \{\text{integers}\} = \{\ldots,-2,-1,0,1,2,\ldots\}$

$\boxed{\mathbb{R}} := \{\text{real numbers}\}$ "$=$" $\longleftrightarrow$

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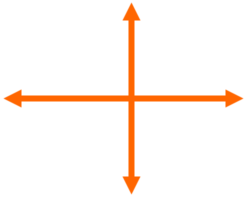

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A **compact interval** is a set of the form $[a,b]$, where $-\infty < a \le b < \infty$.

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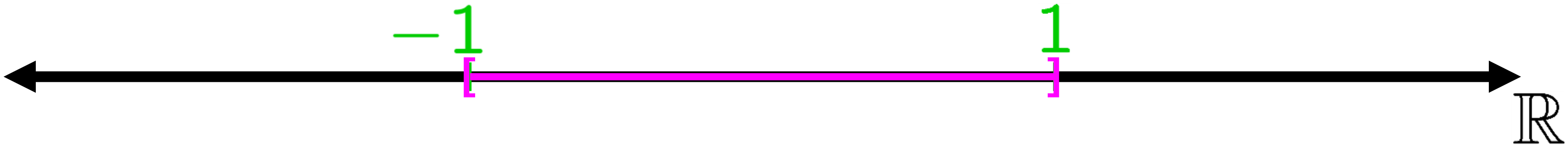
compact interval



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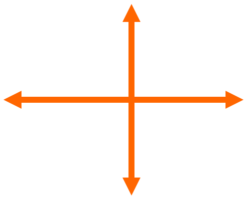

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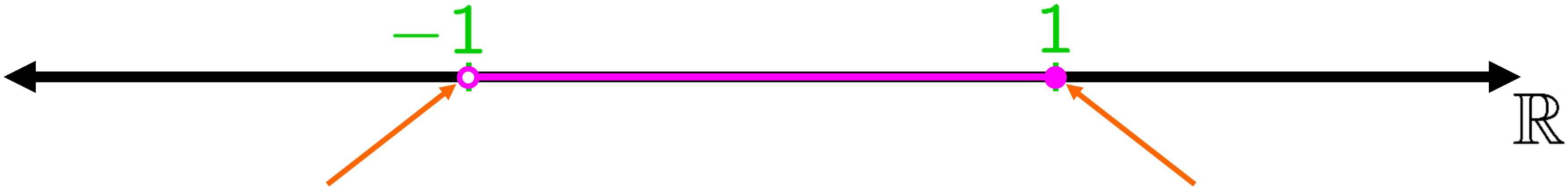
# Standard Notation

$$(-1,1] = \{x \in \mathbb{R} \mid -1 < x \leq 1\}$$

half-open interval

open on left, closed on right



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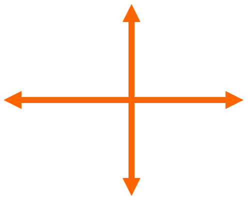

# Standard Notation

$(a, b]$, where $-\infty < a < b < \infty$.

**NOTE**: $(-\infty, 1]$ is NOT half-open.

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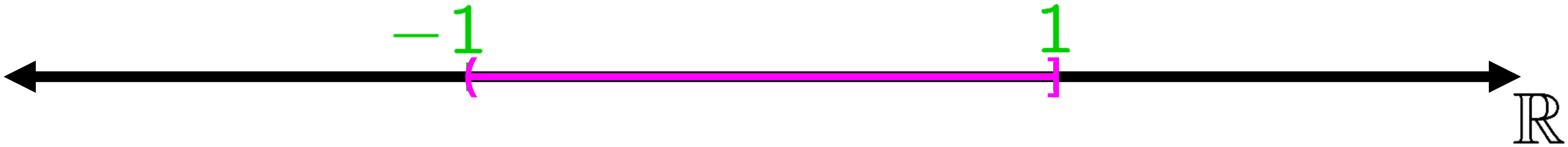

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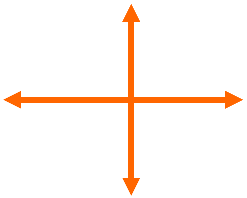
$\boxed{\mathbb{C}} := \{\text{complex numbers}\}$ “=” 

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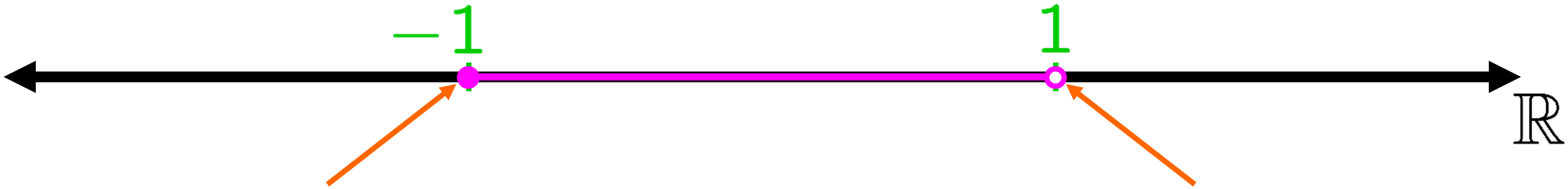

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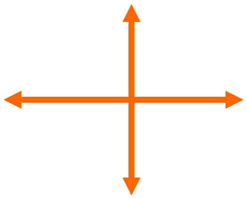

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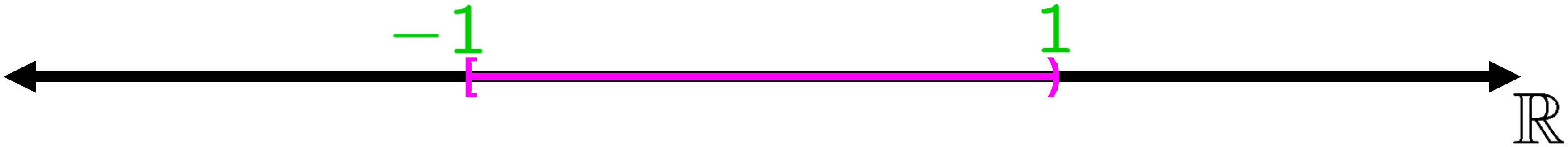

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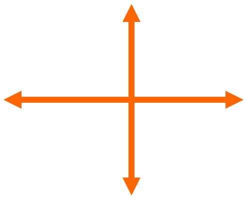

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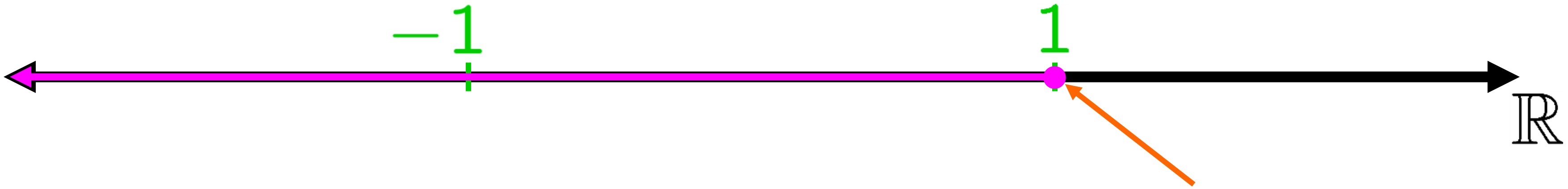

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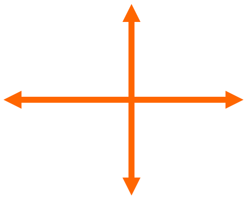
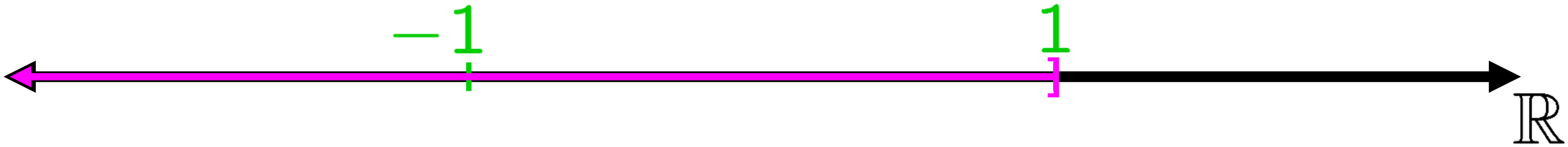


# Standard Notation

Intervals of the form $[a, b]$,
with $-\infty < a < b < \infty$,
are said to be **compact**, *i.e.*, closed and bounded.

$$\boxed{(}-\infty, 1] = \{x \in \mathbb{R} \mid x \leq 1\}$$

$\underbrace{\text{CLOSED}}_{\text{as closed as possible}}$, unbounded interval



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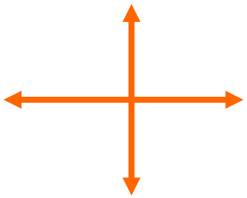

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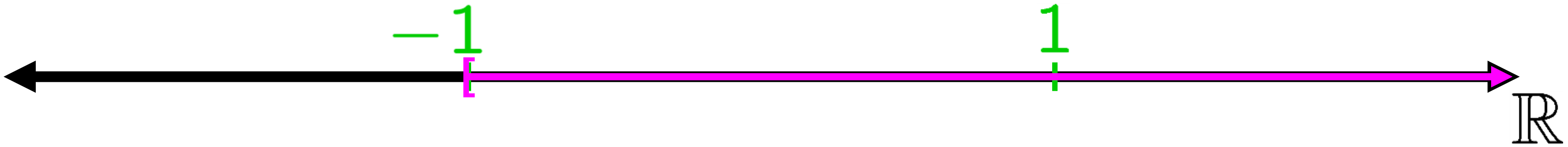

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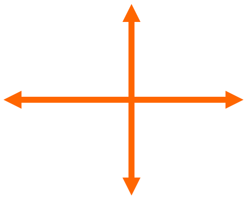


# Standard Notation

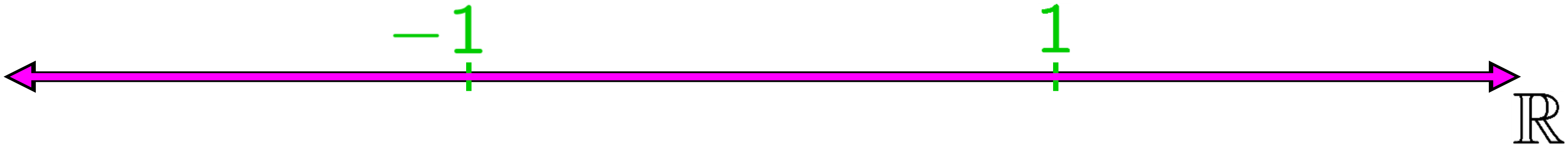
SKILL
identify intvl:
open, closed, half-open, bdd

This is the ONLY interval that is both open and closed.

$$(-\infty, \infty) = \mathbb{R}$$

SKILL
gph interval

$\underbrace{\text{CLOSED}}_{\text{as closed as possible}}$, unbounded interval



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picture?

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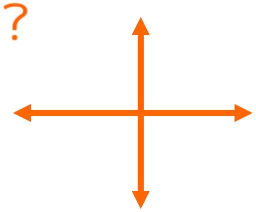
$\boxed{\mathbb{C}} := \{\text{complex numbers}\}$ “=” 

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# Picture $\mathbb{Z}$

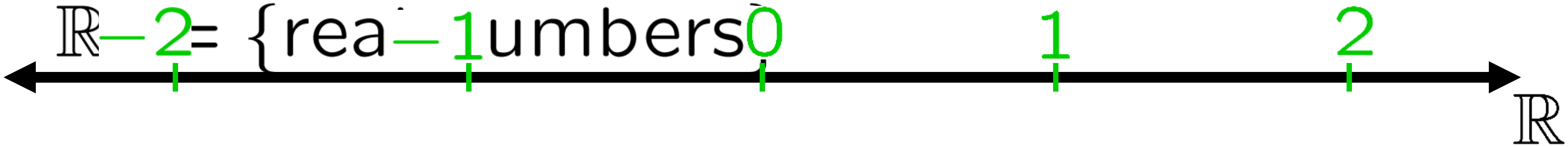
$\mathbb{R} :=$ {real numbers}

$\mathbb{Z} :=$ {integers}

picture?

$\mathbb{Z} :=$ {integers}

$\mathbb{R} :=$ {real numbers}

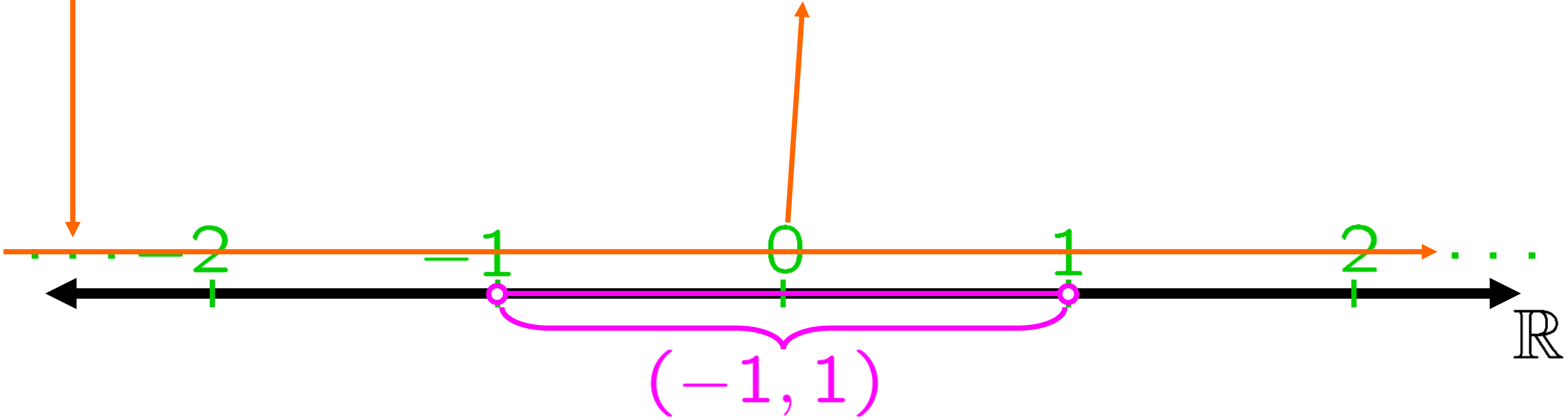


# Picture $\mathbb{Z}$

$\mathbb{R} := \{\text{real numbers}\}$

$\mathbb{Z} := \{\text{integers}\}$

$$\mathbb{Z} \underset{\substack{\in \\ 0}}{\cap} (-1,1) \neq \emptyset$$



# Picture $\mathbb{Z}$

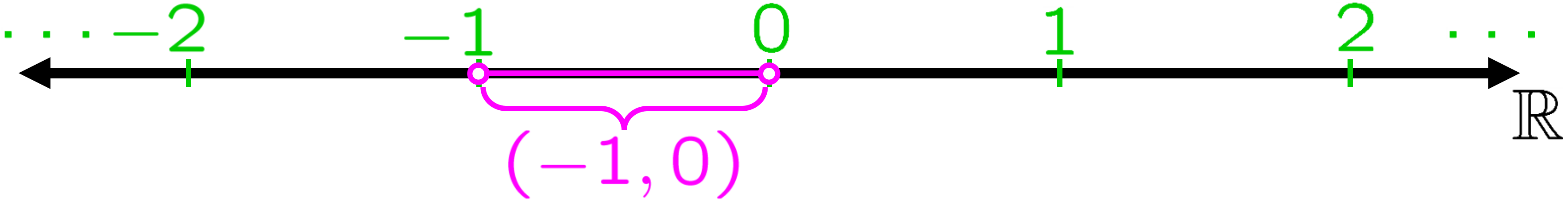
$\mathbb{R} :=$ {real numbers}

$\mathbb{Z} :=$ {integers}

$$\mathbb{Z} \cap (-1, 1) \neq \emptyset$$

$$\mathbb{Z} \cap (-1, 0) = \emptyset$$

$$\mathbb{Z} \cap (\sqrt{2} - 0.001, \sqrt{2} + 0.001) = \emptyset$$



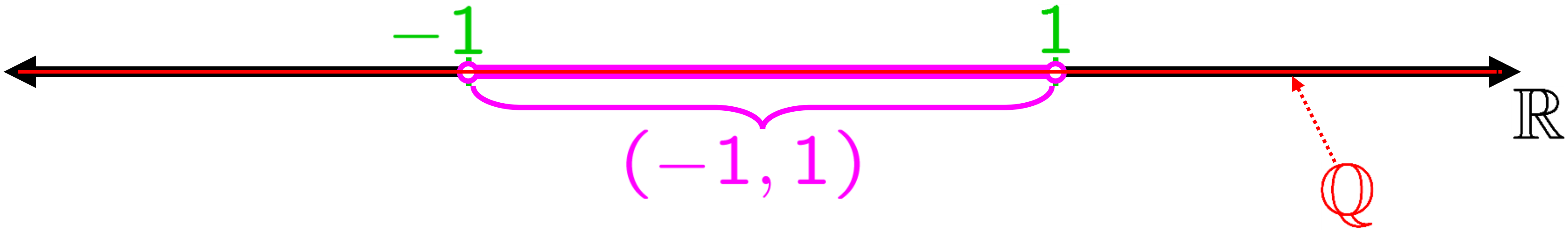
**Picture $\mathbb{Q}$**

$\mathbb{R} :=$ {real numbers}

$$\mathbb{Q} \cap (-1, 1) \neq \emptyset$$

$\mathbb{Q} :=$ {rational numbers}

$\mathbb{Z} :=$ {integers}



# Picture $\mathbb{Q}$

$\mathbb{R} := \{\text{real numbers}\}$

$$\mathbb{Q} \cap (\sqrt{2} - 0.001, \sqrt{2} + 0.001) \neq \emptyset$$

$$\mathbb{Q} \cap (-1, 1) \neq \emptyset$$

$$\mathbb{Q} \cap (-1, 0) \neq \emptyset$$

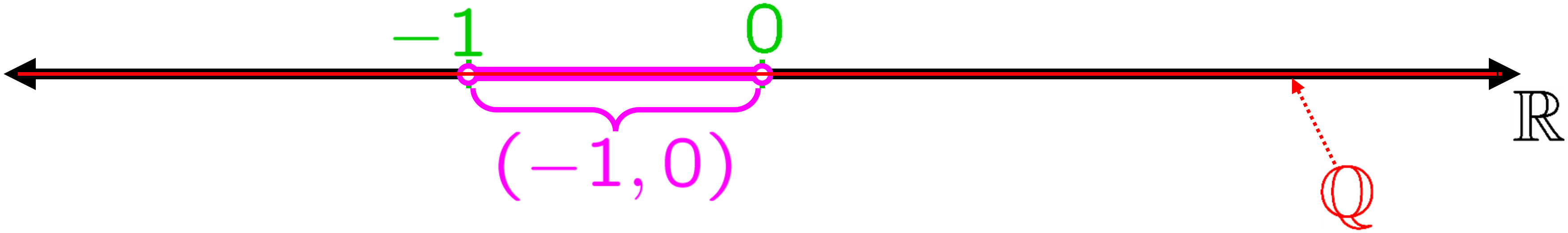
$\mathbb{Q} := \{\text{rational numbers}\}$ is $\boxed{\text{dense}}$ in $\mathbb{R}$.

meets every open interval

has non-$\emptyset$ intersection with

$\mathbb{Z} := \{\text{integers}\}$

STOP



Spp