MATH 1271 LEC 40: PRACTICE MIDTERM2 (SPRING 2011)

Name:____________________________________

Signature:____________________________________

Section: ______________ TA:__________________

Score:

Problem 1.___________________________
Problem 2.___________________________
Problem 3.___________________________
Problem 4.___________________________
Problem 5.___________________________
Problem 6.___________________________
Total:___________________________

Instruction: There are a total of 100 points on this exam. To get full credit for a problem you must show the details of your work. Answers unsupported by by an argument will get little credit. No books, notes, calculators, cell phones or other electronic devices are allowed. Do all of your calculations on this test paper.

Problem 1 (20 points): Compute the following:
(a) (5 points) $f(x) = \log_2(1 + x^2)$. Find $f'(x)$.

(b) (5 points) $f(x) = \frac{(x-2)^2(x^2+1)^3}{x+1}$. Find $f'(x)$.

(c) (5 points) Evaluate $\lim_{x\to 1} \frac{x^a-1}{x^b-1}$, where $a, b \neq 0$.

(d) (5 points) Evaluate $\lim_{x\to\infty} \frac{\ln x}{x}$.

Problem 2 (20 points): Sand falling at the rate of 3 $\mathrm{ft}^3/\mathrm{min}$ forms a conical pile whose base radius r always equals twice the height h. Find the rate at which the height is changing at the instant when the height is 10 feet. Recall that the volume V of a right circular cone is $V = \frac{1}{3}\pi r^2 h$.

Problem 3 (20 points): A radioactive material of initial mass 20 milligrams decays to 5 milligrams after 5 years.

(a)(15 points) Find an expression for the radioactive mass remaining after t years.

(b)(5 points) What is the half-life of the material?

Problem 4 (15 points): Use linear approximation to estimate $(26.96)^{\frac{1}{3}}$.

Problem 5 (5 points): The function $f(x)$ is differentiable on $[0,2]$. Suppose that $2 \leq f'(x) \leq 3$ for all x in $[0,2]$, and $f(0) = 1$. Prove that

$$5 \leq f(2) \leq 7.$$

(Hint: consider $\frac{f(2)-f(0)}{2-0}$.)

Problem 6 (20 points): Find the global maximum and global minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$ on the closed interval $[-2, 3]$. Find the interval that $f(x)$ is concave upward.

Very brief Solutions:

Problem 1. (a) $\frac{2x}{(1+x^2)\ln 2}$ (b) $\frac{a}{b}$ (L'Hospital's Rule)

(c) $\frac{(x-2)^2(x^2+1)^3}{x+1}\left(\frac{2}{x-2}+\frac{6x}{x^2+1}-\frac{1}{x+1}\right)$ (logarithm differentiation)

(d) 0 (L'Hospital's Rule)

Problem 2. $\left.\frac{dh}{dt}\right|_{h=10}=\frac{3}{400\pi}$ft/min

Problem 3. (a) $m(t)=20\left(\frac{1}{4}\right)^{t/5}$ milligrams, or $m(t)=20e^{-\frac{\ln 4}{5}t}$ milligrams.

(b) 2.5 years

Problem 4. $3-\frac{1}{675}$

Problem 5. By mean value theorem, and $f(0)=1$,

$$\frac{f(2)-f(0)}{2-0}=f'(c)\Rightarrow\ 2\le\frac{f(2)-f(0)}{2-0}\le 3\Rightarrow 5\le f(2)\le 7.$$

Problem 6. Global maximum is $f(-1)=7$, global minimum is $f(2)=-20$. The interval that f is concave upward is $(1/2,3]$.