Name: ______________________________

Signature: ______________________________

Section: ______________________________

Math 1271. Lecture 030 Practice Midterm Exam II

There are a total of 100 points on this exam. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit. No books, notes, calculators, cell phones or other elecronic devices are allowed. Do all of your calculations on this test paper.

Problem	Score
1.	
2.	
3.	
4.	
5.	
Total:	

Problem 1 *Compute the indicated derivatives of the functions $y = f(x)$. It is not necessary to simplify.*

a. (5 points) $f'(x)$ *where*

$$f(x) = \frac{\sin(1 - e^x)}{x}.$$

b.(5 points) y' *where*

$$y = (x+3)^{10}(x^2 - 9)^6(x-1)^7.$$

c.(5 points) $f'(x)$ *where*

$$f(x) = x \arcsin(x).$$

d.(5 points) $f^{(7)}(x)$ *where*

$$f(x) = \frac{1}{2x+1}.$$

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Problem 2 *(20 points) Sand falling at the rate of* $3\ \mathrm{ft}^3/\mathrm{min}$ *forms a conical pile whose radius* r *always equals twice the height* h. *Find the rate at which the height is changing at the instant when the height is* 10 *feet. Recall that the volume* V *of a right circular cone is* $V = \frac{1}{3}\pi r^2 h$.

Problem 3 *Let y be a function of x such that $x^2y - y^3 = 1$ and the derivatives y' and y'' exist at $x = 0$.*

a. (15 points) *If $y(0) = -1$, compute $y'(0)$.*

b. (5 points) *Also compute $y''(0)$.*

Problem 4 *(20 points) Use differentials to find an approximation to* $(26.96)^{\frac{1}{3}}$.

Problem 5 *A radioactive material of initial mass 20 milligrams decays to 5 milligrams after 5 years.*

a. 15 points *Find an expression for the radioactive mass remaining after t years.*

b. 5 points *What is the half-life of the material?*

(Very) Brief Solutions:

1a.

$$f'(x) = \frac{-xe^x \cos(1-e^x) - \sin(1-e^x)}{x^2}$$

1b.

$$y' = (x+3)^{10}(x^2-9)^6(x-1)^7 \left[\frac{10}{x+3} + \frac{12x}{x^2-9} + \frac{7}{x-1}\right]$$

1c.

$$f'(x) = \arcsin(x) + \frac{x}{\sqrt{1-x^2}}$$

1d.

$$f^{(7)}(x) = -\frac{2^7(7!)}{(2x+1)^8}$$

2

$$\frac{dh}{dt}|_{h=10} = \frac{3}{400\pi} \text{ ft./min.}$$

3a.

$$y'(0) = 0$$

3b.

$$y''(0) = -2/3$$

4. $3 - 1/675$

5a. $m(t) = 20(\frac{1}{4})^{t/5}$ milligrams, or $m(t) = 20e^{-\frac{\ln 4}{5}t}$ milligrams.

5b. 2.5 years