Name: ______________________________

Signature: ______________________________

Section: ______________________________

Math 1271. Lecture 030   Practice Midterm Exam III

There are a total of 100 points on this 50 minute exam. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit. No books, notes, calculators, cell phones or other elecronic devices are allowed. Do all of your calculations on this test paper.

Problem	Score
1.	________
2.	________
3.	________
4.	________
5.	________
Total:	________

Problem 1 *Find the derivative $f'(x)$ and simplify.*

a. (5 points)

$$f(x) = \ln(e^{x^{-2}})$$

b. (5 points)

$$f(x) = x^{x^4}$$

c. (5 points)

$$f(x) = \int_x^3 \sqrt{1+t^3}\ dt$$

d. (5 points)

$$f(x) = \int_1^{5x^2-1} \sin(t^3)\ dt.$$

Problem 2 *Compute the integrals. Note that some of these integrals are indefinite and some definite.*

a. (5 points)

$$\int_0^{\pi/2} \frac{\cos x \sin x}{3 + \cos^2 x}\, dx$$

b. (5 points)

$$\int \frac{(x+1)^2}{(1-x^2)^2}\, dx$$

c. (5 points)

$$\int_0^2 (1+x)^{3/2}\ dx$$

d. (5 points)

$$\int \frac{e^x}{(e^x+1)^3}\ dx$$

Problem 3 *(20 points)* *Find the area of the finite region in the plane bounded by the curve* $y = -x^2 + x + 2$ *and the* x*-axis.*

Problem 4 *(20 points) The Okefenokee Frozen Orange Juice Company wishes to package frozen orange juice in cylindrical cans of volume* $V = 2,700\ \mathrm{cm}^3$. *The cost per unit area of the cardboard cylindrical part of the can is* $.001$ *cents per* cm^2*, and that of the metal ends is* $.004$ *cents per* cm^2. *What are the proportions of the container of minimum cost? Recall that if the circular base of the cylinder has radius* r *and the height is* h*, the area of the side is* $2\pi rh$ *and the volume is* $\pi r^2 h$.

Problem 5 *(20 points) Given the function*

$$y = f(x) = \frac{1}{x},$$

and the partition $P_3 : 1, 2, 3, 4$ *of the interval* $[1, 4]$.

a *Compute the upper Riemann sum* U_3.

b. *Compute the lower Riemann sum* L_3.

c. *Based on the results of [a.] and [b.] find an estimate of*

$$\int_1^4 \frac{1}{x}\, dx$$

that differs from the true value by at most $3/4$.

Brief solutions:

1a.

$$f'(x) = -\frac{2}{x^3}$$

1b.

$$f'(x) = x^{x^4}(4x^3 \ln x + x^3]$$

1c.

$$f'(x) = -\sqrt{1 + x^3}$$

1d.

$$f'(x) = 10x \sin([5x^2 - 1]^3)$$

2a.

$$\frac{1}{2} \ln \frac{4}{3}$$

2b.

$$\frac{1}{1 - x} + C$$

2c

$$\frac{2}{5}(3^{5/2} - 1)$$

2d.

$$-\frac{1}{2(e^x + 1)^2} + C$$

3.

$$\frac{9}{2}$$

4.

$$r = \frac{15}{(10\pi)^{1/3}} \text{ cm}, \quad h = \frac{120}{(10\pi)^{1/3}} \text{ cm}$$

5a.

$$U_3 = \frac{22}{12}$$

5b.

$$L_3 = \frac{13}{12}$$

5c.

$$\frac{22}{12} > \int_1^4 \frac{1}{x}\,dx > \frac{13}{12}$$

and $22/12 - 13/12 = 3/4$ so any estimate between U_3 and L_3 will do. The midpoint rule gives the estimate $142/105$.