MATH 8702
Homework Assignment #1
Due 02/01

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1. Let f be a holomorphic map on the unit disc $\mathbb{D}$, and $|f(z)| < 1$ for all $z \in \mathbb{D}$. Prove that if f has two fixed points, then $f(z) = z$.
2. Suppose U and V are conformally equivalent. Prove that if U is simply connected, then so is V.
3. Determine the image of the strip $\{z \mid -1 < Im(z) < 1\}$ under the map $f(z) = \dfrac{z}{z+i}$.
4. Find a conformal map between the unit disc and the domain $\{z \mid Re(z) > 0, Im(z) > 0\}$.
5. Let U denote the open connected set between two circles which are tangent at the point z_0. Map U conformally onto the unit disc.
6. Prove that there is no one-to-one conformal map of the punctured disc $\{z \in \mathbb{C} : 0 < |z| < 1\}$ onto the annulus $\{z \in \mathbb{C} : 1 < |z| < 2\}$.
7. Find a conformal map f between the unit disc and the region $\{z \in \mathbb{C} : |arg(z)| < 1\}$ such that $f(0) = 1$.