MATH 4707 MIDTERM II

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INSTRUCTOR: Anar Akhmedov

Name: ______________________________

Signature: ______________________________

ID #: ______________________________

Show all of your work. No credit will be given for an answer without some work or explanation.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total (150 points)	

1. (20 points) Suppose that n is a positive integer such that $2^n + 1$ is prime. Show that n is a power of 2.

2. (30 points) Find the following values

   a) $\phi(100^n)$, where $\phi(m)$ is the number of numbers that are not larger than m and relatively prime to m.

   b) $\tau(100^n)$, where $\tau(m)$ is the number of divisors of m.

   c) $\sigma(100^n)$, where $\sigma(m)$ is the sum of divisors of m.

3. (20 points) Show that the equation $x^2+y^2 = 3(t^2+s^2)$ has no integer solutions other than $x = y = t = s = 0$.

4. (20 points) Prove that a graph G is bipartite if and only if it has no odd cycles.

5. (20 points) Show that every graph G of order $v_G \geq 1$ has at least $e_G - v_G + 1$ cycles. (Hint: Use induction)

6. (20 points) Show that if a tree G has order $n \geq 4$ and not a star, then $\bar{G}$ is connected and has at least $n - 4$ cycles of length 3.

7. (20 points) Prove that at a party of 14 people either there are five mutual acquaintances or there are three mutual strangers.