MATH 8301 TAKE HOME FINAL

December 6, 2011

INSTRUCTOR: Anar Akhmedov

Name: ______________________________

Signature: ______________________________

ID #: ______________________________

Show all of your work. No credit will be given for an answer without some work or explanation. You must do this exam **individually**. You may not consult with any other people about the exam, whether a student in this course or not. If a problem is unclear, you can ask me for a clarification. The exam is to be handed in to me by **2.00pm Friday, December 16**. I'll be holding office hours from 1.00pm-2.00pm on Friday, December 16. The **late** take home final **will not be accepted**.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total (70 points)	

1. (10 points) Given a map $f : \mathbb{S}^{2n} \to \mathbb{S}^{2n}$, show that there is some point $x \in \mathbb{S}^{2n}$ with either $f(x) = x$ or $f(x) = -x$. Deduce that every map $\mathbb{RP}^{2n} \to \mathbb{RP}^{2n}$ has a fixed point. Construct maps $\mathbb{RP}^{2n-1} \to \mathbb{RP}^{2n-1}$ without fixed points from linear transformations $\mathbb{R}^{2n} \to \mathbb{R}^{2n}$ without eigenvectors.

2. (10 points) Compute the relative homology groups $H_i(\mathbb{RP}^n, \mathbb{RP}^m)$ for $m \leq n$.

3. (10 points) Let $f : S^{2n-1} \to S^n$ be a continuous map, and $X_f = D^{2n} \cup_f S^n$ denote the topological space obtained from S^n by attaching a $2n$-cell using f. Compute the homology groups of X_f.

4. (10 points). Let M_g be a closed orientable surface of genus $g \geq 1$. Prove that any continuous map $f : \mathbb{RP}^2 \to M_g$ is null-homotopic.

5. (10 points) Let X be the quotient space of S^2 under the identifications $x \sim -x$ for x in the equator S^1 . Compute the fundamental group and homology groups $H_i(X)$. Do the same for S^3 with antipodal points of the equatorial $S^2 \subset S^3$ identified.

6. (10 points) Let g be a continous map from S^2 to S^2 such that $||g(x) - x|| < 1$ for all x in S^2. Must g be a surjective map? Justify your answer.

7. (10 points) Let $0 \to A_* \to B_* \to C_* \to 0$ be a short exact sequence of chain complexes. Show that if two of the three complexes A_*, B_*, C_* are exact, then so is the third.