# MATH 8301 MIDTERM

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Name: ______________________________

Signature: ______________________________

ID #: ______________________________

Show all of your work. No credit will be given for an answer without some work or explanation.

| Problem | Points |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| Total (50 points) | |

1. (14 points) Construct a CW complex $X$ such that

   a) $\pi_1(X) = \mathbb{Z}_{20} \times \mathbb{Z}_{11}$

   b) $\pi_1(X) = S_3$, where $S_3$ is the group of permutations of $\{1, 2, 3\}$

   Justify your answer.

2. (12 points) Let $X$ be the topological space obtained from $\mathbb{R}^3$ by removing the three coordinate axes. Compute $\pi_1(X)$.

3. (12 points) The graph $\mathbb{G}$ has six vertices $a_1$, $a_2$, $a_3$, $b_1$, $b_2$, $b_3$ and nine edges $a_i b_j$ for $i, j = 1, 2, 3$. Let $X_{\mathbb{G}}$ be a space obtained from $\mathbb{G}$ by attaching a 2-cell along each loop formed by a cycle of four edges in $\mathbb{G}$. Find $\pi_1(X_{\mathbb{G}})$.

4. (12 points) Suppose that $f_t : X \longrightarrow X$ is a homotopy such that $f_0$ and $f_1$ are each the identity map. Show that for any $x_0 \in X$, the loop $f_t(x_0)$ represents an element of the center of $\pi_1(X, x_0)$.