MATH 8302 TAKE HOME FINAL

May 11, 2012

INSTRUCTOR: Anar Akhmedov

Name: ______________________

Signature: ______________________

ID #: ______________________

Show all of your work. No credit will be given for an answer without some work or explanation. You must do this exam **individually**. You may not consult with any other people about the exam, whether a student in this course or not. If a problem is unclear, you can ask me for a clarification. The exam is to be handed in to me by **2.00pm Friday, May 11**. I'll be holding office hours from 1.00pm-2.00pm on Friday, May 11. The **late** take home final **will not be accepted**.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total (70 points)	

1. (10 points) Let $\alpha : \mathbb{S}^n \to \mathbb{S}^n$ be the antipodal map: $\alpha(x) = -x$. Show that α is an orientation-preserving if and only if n is odd.

2. (10 points) Prove that $\mathbb{RP}^n$ is orientable if and only if n is odd.

3. (10 points) Let ω be a closed 2-form on $\mathbb{S}^4$. Show that the 4-form $\omega \wedge \omega$ vanishes at some point of $\mathbb{S}^4$.

4. (10 points). Let $\mathbb{T}^2$ be a 2-dimensional torus, and $\psi : \mathbb{S}^2 \to \mathbb{T}^2$ be any smooth map. Prove that for any cohomology class $[\omega] \in {H_{dR}}^2(\mathbb{T}^2)$, we have $\psi^*([\omega]) = 0$.

5. (10 points) Let $\mathbb{SO}(n)$ denote the group of orthogonal $n \times n$ matrices with real coefficients and determinant 1.

   (a) Show that $\mathbb{SO}(n)$ is a submanifold of $\mathbb{GL}(n, \mathbb{R})$ of dimension $n(n-1)/2$.

   (b) Prove that $\mathbb{SO}(n)$ admits a nowhere zero vector field.

   (c) Compute the Euler characteristic of $\mathbb{SO}(n)$.

6. (10 points) Let $S = \{(x, y, z) \in \mathbb{S}^2 : x^2 z = y^3 - yz^2\}$. Is S a smooth submanifold of $\mathbb{R}^3$? Justify your answer.

7. (10 points) If M is a compact, smooth, oriented manifold with boundary, show that there does not exist a smooth retraction of M onto its boundary.