MATH 8306 TAKE HOME FINAL

December 3, 2010

INSTRUCTOR: Anar Akhmedov

Name: ______________________________

Signature: ______________________________

ID #: ______________________________

Show all of your work. No credit will be given for an answer without some work or explanation. You must do this exam **individually**. You may not consult with any other people about the exam, whether a student in this course or not. If a problem is unclear, you can ask me for a clarification. The exam is to be handed in to me by **noon Monday, December 20th**. For your convenience, I'll be holding office hours from 10.00am-12.00pm on Monday, December 20th. The **late** take home final **will not be accepted**.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total (30 points)	

1. (4 points) Construct a CW complex X such that $H_0(X) = \mathbf{Z}$, $H_3(X) = \mathbf{Z_{21}}$, $H_7(X) = \mathbf{Z} \oplus \mathbf{Z} \oplus \mathbf{Z}$ and $H_n(X) = 0$ for all $n \neq 0, 3, 7$.

2. (4 points) Let G be a topological group (i.e. a group G endowed with a topology that makes multiplication and inversion continous). Show that $\pi_1(G, e)$ is abelian, where e denote the identity element in G.

3. (5 points) Let X be a CW-complex. Show that if $p : RP^{2n} \to X$ is a covering map, then p is a homeomorphism.

4. (5 points) Prove that $\tilde{H}_k(X * Y) \cong H_{k-1}(X \times Y, X \vee Y)$.

5. (3 points) Let $\phi : X \to X$ be a continous map satisfying the following assumptions: $\phi \circ \phi \circ \phi = id_X$ and $\phi(x) \neq x$ for any $x \in X$. Show that there cannot be such a function ϕ if $X = S^2$. What if X is the two-dimensional torus?

6. (4 points) Let K be the Klein bottle. Compute the cohomology groups of K with $\mathbf{Z}$ and $\mathbf{Z_2}$ coefficients.

7. (5 points) Let $\mathbf{SO(n)}$ denote the group of orthogonal $n \times n$ matrices with real coefficients and determinant 1.

   (a) Show that $\mathbf{SO(n)}$ is a manifold.

   (b) Prove that $\mathbf{SO(n)}$ admits a nowhere zero vector field.

   (c) Compute the Euler characteristic of $\mathbf{SO(n)}$.