MATH 8307 TAKE HOME FINAL

April 29, 2016

INSTRUCTOR: Anar Akhmedov

Name: ______________________________

Signature: ______________________________

ID #: ______________________________

Show all of your work. No credit will be given for an answer without some work or explanation. You must do this exam **individually**. You may not consult with any other people about the exam, whether a student in this course or not. If a problem is unclear, you can ask me for a clarification. The exam is to be handed in to me by **3.20pm Thursday, May 12**. For your convenience, I'll be holding office hours from 2.40pm-3.20pm on Thursday, May 12. The **late** take home final **will not be accepted**.

Problem	Points
1	
2	
3	
4	
5	
6	
Total (30 points)	

1. (5 points) Prove the follwing theorem Stiefel. If $n + 1 = 2^r m$ with m odd, then there does not exist 2^r vector fields on the projective space $\mathbb{RP}^n$ which are everywhere linearly independent.

2. (5 points) Compute $\pi_2(\mathbb{CP}^2, \mathbb{RP}^2)$.

3. (5 points) Let M be a compact, orientable 4-manifold with the same betti numbers as $\mathbb{CP}^2$ and $H_1(M) = 0$. Show that all the homology groups of M are torsion free and M has the same cohomology ring as $\mathbb{CP}^2$.

4. (5 points) Show that the set of isomorphism classes of 1-dimensional vector bundles over the space B form an abelian group with respect to the tensor product operation. Compute the above group for $B = S^1$.

5. (5 points) Show that the set Ω_n consisting of all unoriented cobordism classes of smooth closed n-manifolds can be made into an abelian group. Show that Ω_4 contains at least four distinct elements.

6. (5 points) Show that the Hopf invariant of a map $f : S^{4n-1} \to S^{2n}$ defines a homomorphism. Using this homomorphism show that $\pi_{4n-1}(S^{2n}) \cong \mathbb{Z} \oplus A$, where A is some abelian group.