MATH 8702 TAKE HOME FINAL

May 9, 2013

INSTRUCTOR: Anar Akhmedov

Name: ____________________________

Signature: ____________________________

ID #: ____________________________

Show all of your work. No credit will be given for an answer without some work or explanation. You must do this exam **individually**. You may not consult with any other people about the exam, whether a student in this course or not. If a problem is unclear, you can ask me for a clarification. The exam is to be handed in to me by **2.30pm Friday, May 17**. The **late** take home final **will not be accepted**.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total (70 points)	

1. (10 points) Suppose that τ is purely imaginary, say $\tau = it$ with $t > 0$. Consider the division of the complex plane into congurent rectangles obtained by considering the lines $x = n/2$, $y = tm/2$ as n and m range over the integers. (An example is the rectangle whose vertices are 0, $1/2$, $1/2 + \tau/2$, and $\tau/2$).

   a) Show that $\wp$ is a real-valued on all these lines, and hence on the boundaries of all these rectangles.

   b) Prove that $\wp$ maps the interior of each rectangle conformally to the upper (or lower) half-plane.

2. (10 points) Let $\Omega = \mathbb{D} \setminus \{\frac{1}{2}, \frac{-1}{2}\}$. Determine all analytic functions $f : \Omega \to \Omega$ with the following property: if γ is any cycle in Ω which is not homologus to zero $(mod\ \Omega)$, then $f * \gamma$ is not homologus to zero $(mod\ \Omega)$.

3. (10 points) Prove that a continuous function on a domain is harmonic if and only if it satisfies the mean value property.

4. (10 points) By direct computation, show that if $r(z)$ is a rational function of z, then the meromorphic 1-form $r(z)dz$ of the Riemann sphere $\mathbb{C}_\infty$ satisfies the Residue Theorem. (Hint: write $r(z)$ in partial fractions.)

5. (10 points) Let X be the compact Riemann surface associated to the equation $z^{2a} - 2w^b z^a + 1 = 0$, for fixed integers a, b. Identify the branch points of the covering of the Riemann sphere defined by the z coordinate and compute the genus of X.

6. (10 points) Let X be the hyperelliptic surface defined by $y^2 = x^5 - x$. Note that x and y are meromorphic functions on X. Compute the principal divisors $div(x)$ and $div(y)$.

7. (10 points) Show that the "Klein curve" X defined by $xy^3 + yz^3 + zx^3 = 0$ has genus 3, and realizes the Hurwitz bound by finding 168 automorphisms of X.