MATH 2283 SAMPLE MIDTERM I

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The Midterm Exam I will cover the Chapters 1 and 2.

1. Prove that $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n\cdot(n+1)}<1$ for any $n\geq 1$.

2. Use mathematical induction to prove the following inequality: If $x\geq -1$, then $(1+x)^n\geq 1+nx$ for all natural numbers n.

3. For every natural number n, show that $u_n=\frac{(1+\sqrt{5})^n-(1-\sqrt{5})^n}{2^n\sqrt{5}}$ is a natural number.

4. Show that if X and Y are finite sets, then $|X|+|Y|=|X\cup Y|+|X\cap Y|$ (where $|S|$ denotes the number of elements of set S).

5. Let x and y be real numbers satisfying $x<y$. Prove that there exists a rational number between x and y.