MATH 1571H SAMPLE MIDTERM PROBLEMS

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The midterm exam will cover the Sections 1.5 - 1.7, 2.2 - 2.6.

1. Express the area of an equlateral triangle as a function of the length of a side.

2. (a) Represent function $h(x) = \sqrt{x^4 - x}$ as a composition of two functions f and g. (b) Provide the formula for the composition $f(g(x))$ of $f(x) = x^3 + x$ and $g(x) = sin(x)$.

3. Find the limit, if it exists. If the limit does not exist, explain why.

   a) $\lim_{x\to -7} \dfrac{x+7}{x^2-49}$

   b) $\lim_{x\to 0} \dfrac{sin(7x)}{sin(5x)}$

   c) $\lim_{x\to 1} (x-1)^2 sin\left(\dfrac{2}{x-1}\right)$

   d) $\lim_{x\to 0} \left(\dfrac{1}{x\sqrt{x+1}} - \dfrac{1}{x}\right)$

   e) $\lim_{x\to -1} \dfrac{|x+1|}{x+1}$

   f) $\lim_{x\to -\infty} \dfrac{1}{x} sin\left(\dfrac{1}{x}\right)$

4. State the ϵ - δ definition of a limit (see page 70, textbook).

5. Find the vertical asymptotes of the function $f(x) = \dfrac{x-3}{(x^2-9)(x+4)}$

6. For what values of x is f continous?

$$f(x) = \begin{cases} 0 & \textit{if } x \textit{ is rational} \\ 1 & \textit{if } x \textit{ is irrational} \end{cases}$$

7. If $p(x)$ is a polynomial of odd degree, show that the equation $p(x) = 0$ has at least one solution.

8. Verify that the function $f(x) = x^3 - 3x + 2$ satisfies the hypothesis of the Mean Value Theorem on the interval $[-2, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

9. Show that the following function is not differentiable at $x = 0$

$$f(x) = \begin{cases} x & \textit{if } x \textit{ is rational} \\ 0 & \textit{if } x \textit{ is irrational} \end{cases}$$

10. Show that $x^3 - 3x + c$ has at most one root in $[0, 1]$, no matter what c may be.