MATH 1571H SAMPLE MIDTERM II PROBLEMS

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The midterm exam will cover the Sections 3.1 - 3.6, 4.1 - 4.6

1. The sum of two positive numbers is 36. What is the smallest possible value of the sum of their squares?

2. Find the global maximum and global minimum of the function $f(x) = 2x^3 - 3x^2 - 12x$ on the closed interval $[-2, 3]$. Find the interval that $f(x)$ is concave upward.

3. Sand falling at the rate of $3ft^3/min$ forms a conical pile whose radius r always equals twice the height h. Find the rate at which the height is changing at the instant when the height is 10 feet. Recall that the volume V of a right circular cone is $V = \frac{1}{3}\pi r^2 h$.

4. Compute the indicated derivatives of the functions $y = f(x)$.

   a) $f(x) = \dfrac{sec(x)}{1 + tan(x)}$

   b) $f(x) = \dfrac{(x-1)^4}{(x^2+2x)^5}$

   c) $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$

   d) $f(x) = sin(5x)cos(3x)$

5. Let y be a function of x such that $x^2y - y^3 = 1$ and the derivatives y' and y'' exist at $x = 0$.

   a) If $y(0) = -3$, compute $y'(0)$.

   b) Compute $y''(0)$.

6. Find the equation of the tangent line to the curve $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

7. Show that $tan(x) > x$ for $0 < x < \pi/2$.

8. Find, correct to six decimal places, the root of the equation $3cos(x) = x + 1$.

9. Find the dimensions of the isosceles triangle of the largest area that can be inscribed in a circle of radius r.