MATH 2283 SAMPLE MIDTERM II

March 21, 2016

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The midterm exam II will cover the sections 3.1 - 3.8.

1. Let s_n be the sequence defined recursively by $s_1 = 2$, $s_n = \sqrt{6 + s_{n-1}}$. Determine whether the sequence s_n converges or diverges. If s_n converges, compute its limit.

2. Suppose the sequence a_n is monotonic. Prove that a_n converges if and only if it is bounded.

3. Determine whether each of the following sequences converges or diverges. If a sequence converges, compute its limit. If a sequences diverges, state whether it diverges to $+\infty$, $-\infty$, or neither.

   a) $a_n = e^{10/n}$

   b) $a_n = \dfrac{5^{n+3}}{7^n}$

   c) $a_n = n^3 e^{-n}$

   d) $a_n = (1 + \frac{3}{n})^n$

   e) $a_n = \dfrac{n^3 - n^2 - n - 1}{10n^2 + n + 1}$

   f) $a_n = \dfrac{n + cos(n)}{2n - sin(2n)}$

   g) $a_n = n^2(1 - cos(1/n))$

4. Prove that a sequence a_n is Cauchy if and only if it is convergent.

5. Assume that a_n and b_n are Cauchy sequences. Use the definition of Cauchy sequence to show that the sequence c_n defined by $c_n = |a_n - b_n|$ is also Cauchy.