

MATH 2283 SAMPLE MIDTERM II

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INSTRUCTOR: Anar Akhmedov

The midterm exam II will cover the sections 3.1 - 3.8.

1. Let s_n be the sequence defined recursively by $s_1 = 2$, $s_n = \sqrt{6 + s_{n-1}}$. Determine whether the sequence s_n converges or diverges. If s_n converges, compute its limit.
2. Suppose the sequence a_n is monotonic. Prove that a_n converges if and only if it is bounded.
3. Determine whether each of the following sequences converges or diverges. If a sequence converges, compute its limit. If a sequence diverges, state whether it diverges to $+\infty$, $-\infty$, or neither.
 - a) $a_n = e^{10/n}$
 - b) $a_n = \frac{5^{n+3}}{7^n}$
 - c) $a_n = n^3 e^{-n}$
 - d) $a_n = \left(1 + \frac{3}{n}\right)^n$
 - e) $a_n = \frac{n^3 - n^2 - n - 1}{10n^2 + n + 1}$
 - f) $a_n = \frac{n + \cos(n)}{2n - \sin(2n)}$
 - g) $a_n = n^2(1 - \cos(1/n))$
4. Prove that a sequence a_n is Cauchy if and only if it is convergent.
5. Assume that a_n and b_n are Cauchy sequences. Use the definition of Cauchy sequence to show that the sequence c_n defined by $c_n = |a_n - b_n|$ is also Cauchy.