MATH 1272 SAMPLE MIDTERM III PROBLEMS

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The midterm exam will cover the Sections 11.1 - 11.9, 12.1 - 12.3.

1. Determine whether each of the following sequences converges or diverges. If a sequence converges, compute its limit. If a sequences diverges, state whether it diverges to $+\infty$, $-\infty$, or neither. Show your reasoning.

   a) $a_n = \dfrac{\ln(n)}{\sqrt{n}}$

   b) $a_n = (1 + \frac{3}{n})^{5n}$

   c) $a_n = (-1)^n \sqrt[n]{n}$

   d) $a_n = \dfrac{sin(2n)}{1+\sqrt{n}}$

   e) $a_n = \dfrac{e^n - e^{-n}}{e^{-2n} - e^{2n}}$

2. Let a_n be the sequence defined recursively by $a_1 = 0$, $a_{n+1} = \dfrac{1}{4}(1+a_n)$. Determine whether the sequence a_n converges or diverges. If a_n converges, compute its limit.

3. If p is a positive constant, show that the improper integral $\displaystyle\int_1^\infty \frac{dx}{x^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

4. Establish the convergence or divergence of the following series by using the comparision test.

   a) $\sum_{n=1}^{\infty} \dfrac{1}{5^n - n}$

   b) $\sum_{n=2}^{\infty} \dfrac{1}{(\ln(n))^n}$

5. Establish the convergence or divergence of the following series by using the integral test.

   a) $\sum_{n=1}^{\infty} \dfrac{n^2}{e^{-n^3}}$

   b) $\sum_{n=1}^{\infty} \dfrac{1}{n^2 + 6n + 13}$

6. Find a power series repersantation for the following functions and determine the interval of convergence.

   a) $f(x) = \dfrac{1}{(1+x)^2}$

b) $g(x) = \dfrac{1}{(1+x)^3}$

7. Show that if the vectors $u + v$ and $u - v$ are orthogonal, then the vectors u and v must have the same length.

8. Find an equation of the set of all points equidistant from the points $A = (-1, 5, 3)$ and $B = (6, 2, -2)$. Describe the set.