MATH 1571H SAMPLE FINAL PROBLEMS

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The final exam will cover the Sections 1.5, 1.6, 2.2 - 2.6, 3.1 - 3.6, 4.1 - 4.6, 5.2 - 5.4, 6.2 - 6.7, 7.2 - 7.5, 8.1 - 8.5, 9.1 - 9.5.

1. Find the limit, if it exists. If the limit does not exist, explain why.

   a) $\lim_{x\to -1} \dfrac{2x^2+3x+1}{x^2-2x-3}$

   b) $\lim_{x\to 0} \dfrac{sin(sin(sin(x)))}{x}$

   c) $\lim_{x\to 0} \dfrac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$

   d) $\lim_{x\to \infty} \left(\sqrt{9x^2+x}-3x\right)$

2. Use the Mean Value Theorem to prove that $|sin(a)-sin(b) \leq |a-b|$ for all a and b.

3. The difference of two numbers is 20. What is the smallest possible value of the product of these numbers?

4. We want to construct a box whose base length is 3 times the base width. The material used to build the top and bottom cost $10/ft^2$ and the material used to build the sides cost $6/ft^2$. If the box must have a volume of $50ft^3$ determine the dimensions that will minimize the cost to build the box.

5. Compute the integrals. Note that some of these integrals are indefinite and some definite.

   a) $\int \dfrac{sin(x)}{cos(x)^{1/3}}dx$

   b) $\int_0^{1/2} \dfrac{x^2-1}{x^4-1}dx$

   c) $\int sin(x)sec^2(cos(x))dx$

   d) $\int x^2\sqrt{x+2}dx$

6. Use the Fundamental Theorem of Calculus to find the derivatives of the following functions.

   a) $\int_{cos(x)}^{sin(x)} ln(2t+1)dt$

   b) $\int_x^{x^2} e^{t^2}dt$

7. Find the solution of the differential equation $\frac{dy}{dx} = \frac{6x^2}{2y + cos(y)}$ that satisfies the initial condition $y(1) = 0$.

8. Given the function $y = f(x) = 5x^2 + 1$ and the partition of the interval $[-1, 1]$ into 5 subintervals of equal length. Compute the Riemann sum using the left endpoints of these subintervals for your sample points.

9. Find the area of the region enclosed by $x = -y^2 + 10$ and $x = (y - 2)^2$.

10. Find the exact length of the curve $y^2 = x$, where $0 \leq x \leq 1$ and $y \geq 0$.

11. Find the volume of the solid obtained by rotating the region bounded by $x = (y - 2)^2$ and $y = x$ about the line $y = -1$-axis.

12. Set up an integral for the volume of the solid obtained by rotating the circle $x^2 + y^2 = r^2$ about the x-axis. Compute the volume of this solid.

13. A $20ft$ cable weighs $80lbs$ and hangs from the ceiling of a building without touching the floor. Determine the work that must be done to lift the bottom end of the chain all the way up until it touches the ceiling.