MATH 1571H SAMPLE MIDTERM III PROBLEMS

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The midterm exam will cover the Sections 5.2 - 5.4, 6.2 - 6.7, 7.2 - 7.5.

1. Show that $cos(x^2) \geq cos(x)$ for $0 \leq x \leq 1$. Deduce that $\int_0^{\pi/6} cos(x^2)dx \geq \frac{1}{2}$.

2. Compute the integrals. Note that some of these integrals are indefinite and some definite.

   a) $\int_0^{\pi/2} \frac{cos(x)sin(x)}{3 + cos^2(x)} dx$

   b) $\int \frac{(x+1)^2}{(1-x^2)^2} dx$

   c) $\int \frac{e^x}{e^x + 1} dx$

   d) $\int x^3\sqrt{x^2+1}dx$

3. Use the Fundamental Theorem of Calculus to find the derivatives of the following functions.

   a) $\int_x^3 \sqrt{t^3+1}dt$

   b) $\int_1^{5x^2-1} sin(t^3)dt$

   c) $\int_1^{x^4} sec(t)dt$

4. Find the solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ that satisfies the initial condition $y(0) = -3$.

5. Given the function $y = f(x) = 1/x$ and the partition $P_3 : 1, 2, 3, 4$ of the interval $[1, 4]$. Compute the upper Riemann sum U_3 and the lower Riemann sum L_3.

6. Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

7. Find the exact length of the curve $y^2 = 4(x + 4)^3$, where $0 \leq x \leq 2$ and $y > 0$.

8. Find the volume of the solid obtained by rotating the region bounded by the curves $y = \frac{1}{4}x^2$ and $y = 5 - x^2$ about the x-axis.

9. Set up an integral for the volume of the solid torus (the donut-shaped solid) with radii r and R obtained by rotating the circle $(x - R)^2 + y^2 = r^2$ about the y-axis. Compute the volume of the torus.

10. Let S be a solid with a circular base of radius r. Parallel cross-sections of S perpendicular to the base are equilateral triangles. Find the volume of the solid S.