

# MATH 1571H SAMPLE MIDTERM PROBLEMS

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The midterm exam will cover the Sections 1.5 - 1.7, 2.1 - 2.6.

- Express the area of an equilateral triangle as a function of the length of a side.
- (a) Represent function  $h(x) = \sqrt{x^4 - x}$  as a composition of two functions  $f$  and  $g$ . (b) Provide the formula for the composition  $f(g(x))$  of  $f(x) = x^3 + x$  and  $g(x) = \sin(x)$ .
- Find the limit, if it exists. If the limit does not exist, explain why.
  - $\lim_{x \rightarrow -7} \frac{x + 7}{x^2 - 49}$
  - $\lim_{x \rightarrow 0} \frac{\sin(7x)}{\sin(5x)}$
  - $\lim_{x \rightarrow 1} (x - 1)^2 \sin\left(\frac{2}{x - 1}\right)$
  - $\lim_{x \rightarrow 0} \left(\frac{1}{x\sqrt{x+1}} - \frac{1}{x}\right)$
  - $\lim_{x \rightarrow -1} \frac{|x + 1|}{x + 1}$
  - $\lim_{x \rightarrow -\infty} \frac{1}{x} \sin\left(\frac{1}{x}\right)$
- State the  $\epsilon$  -  $\delta$  definition of a limit (see page 70, textbook).
- Find the vertical asymptotes of the function  $f(x) = \frac{x - 3}{(x^2 - 9)(x + 4)}$
- For what values of  $x$  is  $f$  continuous?

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

- If  $p(x)$  is a polynomial of odd degree, show that the equation  $p(x) = 0$  has at least one solution.
- Verify that the function  $f(x) = x^3 - 3x + 2$  satisfies the hypothesis of the Mean Value Theorem on the interval  $[-2, 2]$ . Then find all numbers  $c$  that satisfy the conclusion of the Mean Value Theorem.

9. Show that the following function is not differentiable at  $x = 0$

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$