MATH 1572H SAMPLE MIDTERM II PROBLEMS

November 2, 2016

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The midterm exam will cover the Sections 11.1 - 11.3, 12.1 - 12.4, 13.1 - 13.6.

1. Find the limit, if it exists. If the limit does not exist, explain why.
   a) $\lim_{x\to 0} \dfrac{e^x - sin(x) - 1}{x^2 - x^3}$
   b) $\lim_{x\to 0^+} (e^x - 1)^x$

2. Determine whether each of the following sequences converges or diverges. If a sequence converges, compute its limit. If a sequences diverges, state whether it diverges to $+\infty$, $-\infty$, or neither. Show your reasoning.
   a) $a_n = \dfrac{\ln(n)}{\sqrt{n}}$
   b) $a_n = (1 + \frac{3}{n})^{5n}$
   c) $a_n = (-1)^n \sqrt[n]{n}$

3. Let a_n be the sequence defined recursively by $a_1 = 0$, $a_{n+1} = \dfrac{1}{4}(1+a_n)$. Determine whether the sequence a_n converges or diverges. If a_n converges, compute its limit.

4. If p is a positive constant, show that the improper integral $\displaystyle\int_1^\infty \frac{dx}{x^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

5. Establish the convergence or divergence of the following series by using the comparision test.
   a) $\sum_{n=1}^{\infty} \dfrac{1}{5^n - n}$
   b) $\sum_{n=2}^{\infty} \dfrac{1}{(\ln(n))^n}$

6. Establish the convergence or divergence of the following series by using the integral test.
   a) $\sum_{n=1}^{\infty} \dfrac{n^2}{e^{-n^3}}$
   b) $\sum_{n=1}^{\infty} \dfrac{1}{n^2 + 6n + 13}$

7. Use Pappus's theorem to find the volume of the torus (doughnut) generated by revolving a circle of radius r about a line in its plane at a distance R from its center, where $R > r$.