

# MATH 1572H SAMPLE MIDTERM III PROBLEMS

April 22, 2018

INSTRUCTOR: Anar Akhmedov

The midterm exam will cover the Sections 13.7, 13.8, 14.1 - 14.4, 14.6, 15.1 - 15.4, 16.1, 16.3, 16.2, 16.4, 16.5

1. Find the radius of convergence and interval of convergence of the series  $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$
2. Determine whether each of the following series converges or diverges. If it is convergent, find its sum. Show your reasoning.
  - a)  $\sum_{n=0}^{\infty} \frac{3^n}{\pi^n}$
  - b)  $\sum_{n=1}^{\infty} \arctan(n)$
  - c)  $\sum_{n=1}^{\infty} \frac{3}{n(n+1)}$
3. Determine whether the given series converges absolutely, converges conditionally, or diverges. Show your reasoning.
  - a)  $\sum_{n=1}^{\infty} (-1)^n \frac{2^n n!}{n^n}$
  - b)  $\sum_{n=1}^{\infty} (-1)^n \sin^2(1/n)$
4. Establish the convergence or divergence of the following series by using the comparison test.
  - a)  $\sum_{n=1}^{\infty} \frac{1}{5^n - n}$
  - b)  $\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^n}$
5. Establish the convergence or divergence of the following series by using the integral test.
  - a)  $\sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}}$
  - b)  $\sum_{n=1}^{\infty} \frac{1}{n^2 + 6n + 13}$
6. Use power series to solve the differential equation  $y'' = xy'$ .
7. Identify the type of conic section whose equation is  $x^2 = 4y - 2y^2$ . Find the vertices and foci.
8. Find the area enclosed by one loop of the six-leaved rose  $r^2 = 2\cos(3\theta)$ .
9. Find the area that lies outside  $r = 2\sin(\theta)$  and inside  $r = 2\sin(2\theta)$ .

10. Find the length of the polar curve  $r = \theta$ ,  $0 \leq \theta \leq 1$ . Sketch the given curve.