MATH 2283
Sample Midterm Problems

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1. Prove that for any natural number n,
$$2(\sqrt{n+1}-1) < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} < 2\sqrt{n}.$$

2. Use mathematical induction to prove that $7^{2n} - 48n - 1$ is divisible by 2304 for every natural number n.

3. The Fibonacci sequence F_n is defined recursively as follows: $F_1 = F_2 = 1$, $F_n = F_{n-1} + F_{n-2}$. Prove that every fifth Fibonacci number is divisible by 5.

4. Show that if X is a finite set with n elements, then the number of distinct subsets of X is 2^n.

5. Prove that for every positive real number x, there is some positive integer n such that $0 < \frac{1}{n} < x$.