

**MATH 2283**  
**Sample Midterm Problems**

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1. Prove that for any natural number  $n$ ,

$$2(\sqrt{n+1} - 1) < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} < 2\sqrt{n}.$$

2. Use mathematical induction to prove that  $7^{2n} - 48n - 1$  is divisible by 2304 for every natural number  $n$ .
3. The Fibonacci sequence  $F_n$  is defined recursively as follows:  $F_1 = F_2 = 1$ ,  $F_n = F_{n-1} + F_{n-2}$ . Prove that every fifth Fibonacci number is divisible by 5.
4. Show that if  $X$  is a finite set with  $n$  elements, then the number of distinct subsets of  $X$  is  $2^n$ .
5. Prove that for every positive real number  $x$ , there is some positive integer  $n$  such that  $0 < \frac{1}{n} < x$ .