Math 2263: Practice problems for Midterm 2

Problem 1. (15 points) Find the area of the region bounded by the lines $y = -x$ and $y = 1$ and the curve $x = \sqrt{y}$.

Problem 2. Let D be the portion of the circular disk of radius 2 and center at $(0, 0)$ that lies in the upper half of xy-plane. Find the double integral

$$\iint\limits_D e^{-\frac{1}{2}(x^2+y^2)} dA.$$

Problem 3. (5+15 points) Sketch the region D of integration in the following double integral $\int_0^{\pi} \int_{y^2}^{\pi^2} y \cos(x^2) dx dy$. Evaluate the given integral by reversing the order of integration.

Problem 4. (20 points) Let $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2 + 2$. Find the critical points of $f(x, y)$. Use the second derivative test to determine the local maximum, local minimum, and saddle points of $f(x, y)$.

Problem 5. (20 points) Use the method of Lagrange multipliers to find the points on the surface $y^2 = 1 + xz$ that are closest to the origin.

Problem 6. (15+5 points) Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = xy$ subject to the constraint $x^2 + 2y^2 = 1$. Find the extreme values of f on the region $D = \{(x, y) : x^2 + 2y^2 \leq 1\}$.

Problem 7. (10+10 points) Find the volume the ball $E = \{(x, y, z) : x^2+y^2+z^2 \leq R^2\}$ using (i) cylindrical coordinates and (ii) spherical coordinates.

Problem 8. (20 points) Let D be the lamina bounded by the curves $y = \sqrt{x}$ and $y = x^3$. If $\rho(x, y) = x^2 y$ is the density of mass at the point (x, y) in D, then find the y-component of the center of mass of D.

Problem 9. (15 points) Find the area of the surface of the part of the plane $2x+y+z = 2$ that lies in the first octant.

Problem 10. (20 points) Let E be the solid region bounded above by the paraboloid $z = 6 - x^2 - y^2$ and below by the paraboloid $z = x^2 + y^2$. Compute the volume of E.

Problem 11. (20 points) Find the volume of the solid region E which is bounded by the cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$.

Problem 12. (15 points) Evaluate the integral.

$$\int_{-1}^{0} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{1-x^2-y^2}} \frac{1}{x^2 + y^2 + z^2} dz dy dx.$$

[Hint: it is helpful to do a suitable change of coordinates.]