

Math 5654 Assignment 7

This assignment is due Tuesday, May 5. It consists of four problems. From the text, please do Problems 6.2.4, 6.2.6 and 6.2.7. Also, please solve the following problem.

Special stochastic calculus problem

Prove Lemma 1, stated below.

Introductory discussion

We often have to solve simple stochastic integral equations. The method we will use is the familiar *integrating factor* approach. We will need some form of product rule. Let (W_t) be a Wiener process. When we say

$$dX_t = f(t)dt + g(t)dW_t,$$

we simply mean that

$$X_t - X_0 = \int_0^t f(s) ds + \int_0^t g(s) dW_s.$$

(In our course we assume that g is continuously differentiable, so that the integral against the Wiener process is defined. The function f is only assumed to be continuous.)

Lemma 1 (Special product rule for stochastic differentials) *Let (W_t) be a Wiener process and let $dX_t = f(t) dt + g(t) dW_t$, $dY_t = h(t) dt$. Then*

$$d(XY)_t = X_t dY_t + Y_t dX_t. \tag{1}$$

In proving this lemma, to avoid long expressions you may wish to use (and justify) the fact that it is sufficient to prove the lemma in two separate cases: when f is zero and when g is zero.