Some notes about using the method of Undetermined Coefficients:

Let's say your are given the following differential equation:

$$ax''(t) + bx'(t) + cx(t) = f(t)$$

Where $f(t)$ is a function depending on t. We are going to try to find a solution $H(t) + p(t)$ where $H(t)$ is the general solution you all know and love to the homogeneous equation, and $p(t)$ is the particular solution that depends on what $f(t)$ is.

Step 1 First find the general solution for the homogeneous equation. This means find the general solution for:

$$ax''(t) + bx'(t) + cx(t) = 0$$

You will do this in the standard way of finding roots of the polynomial $ay^2 + by + c = 0$. Lets call this solution $H(t)$.

IMPORTANT: If you have initial condition DO NOT solve for c at this point!

Step 2 Now lets figure out what $p(t)$ is. Look at the right hand side. Currently (in class) we are considering three types of $f(t)$. Namely, exponentials, trigonometric functions, and quadratic polynomials. So:

$$f(t) = K \exp^{bt} \ \textit{or} \ K \sin(rt) + L \cos(pt) \ \textit{or} \ \alpha t^2 + \beta t + \gamma$$

Depending on what $f(t)$ is, we get the following "buddy" functions:

$$f(t) = K \exp^{bt} \rightarrow p(t) = A \exp^{bt}$$

$$f(t) = K \sin(rt) + L \cos(pt) \rightarrow p(t) = A \sin(rt) + B \cos(pt)$$

$$f(t) = \alpha t^2 + \beta t + \gamma \rightarrow p(t) = At^2 + Bt + c$$

Notice that if you have sin or cos by itself (if K or L is zero), your $p(t)$ will still be $p(t) = A \sin(rt) + B \cos(pt)$.

Step 3 Now differentiate $p(t)$ to find $p'(t)$ and $p''(t)$. For example, if your $p(t) = A \exp^{bt}$, then $p'(t) = bA \exp^{bt}$ and $p'(t) = b^2 A \exp^{bt}$.

Step 4 Now plug your $p''(t)$, $p'(t)$, and $p(t)$ back into your original differential equation to get:

$$ap''(t) + bp'(t) + cp(t) = f(t)$$

At this point you should be able to solve for the constants A, B, C in your $p(t)$.

Final Step Now you can write down the general solution to the equation which is $H(t) + p(t)$. If the problem has initial conditions and you need to find a specific solution, now is the time to solve for the constants.