03/08/2001 **NAME:**

MATH 1151 QUIZ-7 (18 minutes)

1. (3 points) Solve $\tan 2\theta + 2\cos\theta = 0$ on the interval $0 \le \theta < 2\pi$.

Solution: $\tan 2\theta + 2\cos\theta = \frac{\sin 2\theta}{\cos 2\theta} + 2\cos\theta = \frac{\sin 2\theta + 2\cos 2\theta\cos\theta}{\cos 2\theta} = 0$. This equation is equal to 0 if and only if $\sin 2\theta + 2\cos 2\theta\cos\theta = 0$ and $\cos 2\theta \neq 0$. Now $\sin 2\theta + 2\cos 2\theta\cos\theta = 2\sin\theta\cos\theta + 2(1 - 2\sin^2\theta)\cos\theta = 2\cos\theta(\sin\theta + 1 - 2\sin^2\theta) = 0$. So solution set comes from solving $\cos\theta = 0$ which has $\frac{\pi}{2}, \frac{3\pi}{2}$ as solutions and $\sin\theta + 1 - 2\sin^2\theta = 0$. Now observe that $\sin\theta + 1 - 2\sin^2\theta = 0$ is a quadratic equation in variable $\sin\theta$. So if you use the quadratic root formula you see that roots are $\sin\theta = \frac{-1}{2}$ and $\sin\theta = 1$. Solutions for the first are $\frac{7\pi}{6}, \frac{11\pi}{6}$ and for the second is $\frac{\pi}{2}$. So the solution set is $\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}$. One can easily see that $\cos 2\theta \neq 0$ for these values. **Q.E.D.**

2.(3 points) Find the exact value of $\sin(2\cos^{-1}(\frac{3}{5}))$.

Solution: Now $\cos^{-1}(\frac{3}{5}) = \theta$, so problem is then finding $\sin 2\theta = 2\sin\theta\cos\theta$. If $\cos^{-1}(\frac{3}{5}) = \theta$ then $\cos\theta = \frac{3}{5}$ where $\theta \in [0, \pi)$. By using right triangle method one can see that $\sin\theta = \frac{4}{5}$. So the answer is $2 * \frac{4}{5} * \frac{3}{5} = \frac{24}{25}$. **Q.E.D.**

3.(1 points) A right triangle contains a 25^o angle. If one leg is of length 5 inches, what is the length of the hypotenuse?

Solution: This problem has two solutions depending on where you put angle and side, they are both acceptable, so don't worry. One answer is $\frac{5}{\sin 25^o}$ where side is opposite of the angle. **Q.E.D.**

4.(3 points) Solve the triangle $\alpha = 40^o, \beta = 20^o, a = 2$.

Solution: From the Sinus Law we find that $b = \frac{a * \sin\beta}{\sin\alpha} = \frac{2 * \sin 20^0}{\sin 40^o}$, also $\gamma = 180^o - (20^o + 40^o) = 120^o$, so $c = \frac{a * \sin\gamma}{\sin\alpha} = \frac{2 * \sin 120^o}{\sin 40^o}$. **Q.E.D.**