MATH 1151 MIDTERM EXAM (I), SPRING 2001

Name ______________________ Discussion Section ______________
ID ______________________

(1) (15 points)
Find the exact value of $\sin(\theta)$ and $\tan(\theta)$ for the angle θ which verifies

$$\sec(\theta) = -5/3, \quad \tan(\theta) < 0.$$

Solution: Since $\sec\theta$ is negative and $\tan\theta < 0$. we are in second quadrant. Since $\sec\theta$ is $\frac{1}{\cos\theta}$ we obtain $\cos\theta = -\frac{3}{5}$. Since we are in the second quadrant $\sin\theta$ has to be positive, so by using Pythagorean identity we get $\sin\theta = \frac{4}{5}$. Now since $\tan\theta = \frac{\sin\theta}{\cos\theta}$ we get $\tan\theta = -\frac{4}{3}$. **Q.E.D.**

(2) (24 points, 12 points each)
Find the exact value of the following expressions

$$\text{a) } 6\cos\left(\frac{3\pi}{4}\right) + 2\tan\left(-\frac{10\pi}{3}\right); \quad \text{b) } \sin^2(25^0) + \sin^2(-65^0).$$

Solution: a) $\frac{3\pi}{4}$ is in second quadrant and it is equal to $\frac{\pi}{2} + \frac{\pi}{4}$. So that means it is equal to $-\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$. Now tan is an odd function and has period π. So $\tan -\frac{10\pi}{3} = -\tan\frac{10\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$. So the answer is $-6\frac{\sqrt{2}}{2} + -2\sqrt{3} = -3\sqrt{2} - 2\sqrt{3}$ **Q.E.D.**

b) Since sin is an odd function $\sin -65^o = -\sin 65^o$. But when we take the square negative sign will disappear, so this identity is equal to $\sin^2 25^o + \sin^2 65^o$. Since 25^o and 65^o are complementary angles, sin and cos are complementary triogonometric functions we see that $\sin 65^o = \cos 25^o$. So identity is equal to $\sin^2 25^o + \cos^2 25^o$ which is equal to 1 by Pythagorean Identity. **Q.E.D.**

(3) (18 points)
Let θ be an acute angle such that $\sin(\theta) = 3/4$ and $\cos(\theta) = \sqrt{7}/4$. Find the exact value of

$$\sin(\theta + \pi/2); \quad \sin(\theta + \pi); \quad \sin(\theta + 2\pi).$$

Solution: For θ an acute angle $\sin(\theta+\pi/2) = \cos\theta = \sqrt{7}/4$, $\sin(\theta+\pi) = -\sin\theta = -\frac{3}{4}$, $\sin(\theta + 2\pi) = \sin\theta = \frac{3}{4}$ since sin has period 2π. **Q.E.D.**

(4) (16 points)
What is the angular velocity of the minute hand of a clock? If the minute hand is 2 inches long, how far does its tip move in 20 minutes?
Solution: Now we have the formula that $s = r\theta$. Now $\theta = 2\pi\frac{120}{360}$ since in 20 minutes the hand moves 120^o degrees. So $s = 2 * 2\pi\frac{120}{360} = \frac{4\pi}{3}$ is the amount that hand moves. Now angular velocity is given as $w = \frac{\theta}{t}$ where θ is in radians and t is in minutes. Since minute hand moves 2π in radian in 60 minutes the answer is $w = \frac{2\pi}{60} = \frac{\pi}{30}$. **Q.E.D.**

(5) (27 points)
Find the amplitude, period and phase shift of the function

$$y = 3\sin(2x + \pi/2).$$

Graph this function over two periods.
Solution: I will not graph this function. But I will give some clues. First of all $A = 3$, $T = \frac{2\pi}{2} = \pi$, $\theta = -\frac{\frac{\pi}{2}}{2} = -\frac{\pi}{4}$. Since phase shift is negative that means you have to shift the graph towards left by $\frac{\pi}{4}$, that's one thing to observe, easiest way to check your answers is to see what happens 0 after transformation. In the final form of the graph 0 has to move to $-\frac{\pi}{4}$. Don't forget that graph shrinks by π. **Q.E.D.**