MATH 1151 MIDTERM EXAM (III), SPRING 2001

Name ______________________ Discussion Section ______________
ID ______________________

(1) (16 points)
A triangle has sides of length $a = 9$, $b = 7$ and $c = 10$. Find:
a) the area of the triangle;
b) the measure α of the angle which is opposite to the side of length $a = 9$.

Solution:a) Using Heron's formula and seeing that $s = \frac{9+7+10}{2} = 13$ we see that the answer is $A = \sqrt{13 * (13 - 9) * (13 - 7) * (13 - 10)} = 6\sqrt{26}$.**Q.E.D.**

b) Using Cosine Law we see that $9^2 = 7^2 + 10^2 - 2 * 7 * 10 * \cos\alpha$, form this you can see that $\cos\alpha = \frac{68}{140}$. You can find the α from your calculator by this qeuation easily. **Q.E.D.**

(2) (16 points)
Find at least two different polar coordinates for the point P whose rectangular coordinates are $P = (x, y) = (2, -2\sqrt{3})$.

Solution: We see that $r = 4$ or $r = -4$, by using our equatins for polar coordinates we that $(4, -\frac{\pi}{3})$ and $(-4, \frac{2\pi}{3})$ works. **Q.E.D.**

(3) (18 points)
Write the polar equation $r = -2\cos\theta$ (r and θ represent polar coordinates) using rectangular coordinates (x, y) and sketch the graph of the polar equation.

Solution: Multiplying both sides by r gives us the equation $r^2 = -2x$, so $x^2 + y^2 + 2x = 0$ is our equation. So equation is $(x+1)^2 + y^2 = 1$ which is a circle with radius 1 and center at $(-1, 0)$.**Q.E.D.**

(4) (18 points)
Given the polar equation

$$r = \frac{\sin\theta}{2 + \sin\theta}$$

(r and θ represent polar coordinates) establish the following:
a) does the point P whose polar coordinates are $P = (\frac{1}{3}, \frac{\pi}{6})$ satisfy the equation?
b) from the form of the polar equation can you say whether the graph is symmetric with respect to the line $\theta = \pi/2$ (y-axis)? Why?

Solution: a) This question is really tricky,nobody in class could solve and I have to admit I also did mistake, sorry for the first wrong grading, anyway;putting $P = (\frac{1}{3}, \frac{\pi}{6})$ into the equation we see that we don't get equality but if you put other form of this point which is $P = (-\frac{1}{3}, \pi + \frac{\pi}{6})$ we see that it actually satisfies the equation, if you change the equation into rectangular coordinates you can also see that indeed this point satisfies the equation. **Q.E.D.**

Solution: b) Putting $\pi - \theta$ instead of θ in the equation we see that it is symmetric w.r.t. y-axis since $\sin \pi - \theta = \sin\theta$. **Q.E.D.**

(5) (15 points, 5 each)
Write the following complex numbers in standard form

$$\text{a) } \frac{2+3i}{1-i}; \quad \text{b) } (\overline{4+5i})(7-2i); \quad \text{c) } \sqrt{(2+i)(i-2)}$$

Solution: a) By multiplying with the conjugate of the denominator both numerator and denominator we see that it is equal to $-\frac{1}{2}+\frac{5}{2}i$. **Q.E.D.**

b) $(\overline{4+5i})(7-2i) = (4-5i)(7-2i) = 18-43i$ **Q.E.D.**

c) $\sqrt{(2+i)(i-2)} = \sqrt{-5} = \sqrt{5}i$.**Q.E.D.**

(6) (17 points)
Solve in the complex number system the following equation

$$x^2 - 4x + 7 = 0$$

Solution: By using the quadratic root formula we see that the roots are $\frac{4+\sqrt{16-28}}{2} = 2+\sqrt{3}i$ and $\frac{4-\sqrt{16-28}}{2} = 2-\sqrt{3}i$. **Q.E.D.**