MATH 1151 PRACTICE EXAM, SPRING 2001

(1)
Let P be a point on the edge of a 33rpm record of radius 5 inches.
a) What is the linear speed of P?
b) How many inches the point P travels in 45 seconds?

(2)
Find the exact value of the following trigonometric expression

$$\sin \pi/4 + \tan(-\pi/3).$$

Solution: $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, $\tan -\frac{\pi}{3} = -\tan \frac{\pi}{3} = -\sqrt{3}$, so the answer is $\frac{\sqrt{2}}{2} - \sqrt{3}$. **Q.E.D.**

(3)
Let α be an acute angle such that $\cos \alpha = 1/3$. Find the exact value of $\tan(3\pi/2 - \alpha)$.

Solution: $\sin\left(\frac{3\pi}{2} - \alpha\right) = -\cos \alpha$ by addition formulas for sin function, $\cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin \alpha$ by addition formulas for cos function, so $\tan\left(\frac{3\pi}{2} - \alpha\right) = \frac{-\cos \alpha}{-\sin \alpha} = \cot \alpha = \frac{1}{2\sqrt{2}}$ by looking at the triangle. **Q.E.D.**

(4)
Let α be an angle such that $\cos \alpha = 1/5$ and $\sin \alpha < 0$. Find the exact value of $\csc \alpha$.

Solution: $\sin \alpha = -\frac{\sqrt{24}}{5}$ by the right triangle and this way we see that $\csc \alpha = -\frac{5}{\sqrt{24}}$. **Q.E.D.**

(5)
Find period and phase-shift of the function $f(t) = 2\sin(3t - \pi/2)$.

Solution:$f(t) = 2\sin(3t - \pi/2) = 2\sin(3(t - \pi/6))$, so period is $\frac{2\pi}{3}$. **Q.E.D.**

(6)
Establish the identity

$$\frac{\sec\theta - \cos\theta}{\sec\theta + \cos\theta} = \frac{\sin^2\theta}{1+\cos^2\theta}$$

Solution: $\frac{\sec\theta-\cos\theta}{\sec\theta+\cos\theta} = \frac{\frac{1-cos^2\theta}{\cos\theta}}{\frac{1+cos^2\theta}{\cos\theta}}$, so knowing that $1 - cos^2\theta = \sin^2\theta$, we see that equality holds. **Q.E.D.**

(7)
Establish the identity

$$-2\cot 2\theta = \tan\theta - \cot\theta$$

Solution: Start from the right side, $\tan\theta - \cot\theta = \frac{sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$, since $\sin^2\theta - \cos^2\theta = -\cos 2\theta$ and $2\sin\theta\cos\theta = 2\sin 2\theta$ we get the left side. **Q.E.D.**

(8)
Let α be an angle such that $\pi/2 < \alpha < \pi$ and $\sin\alpha = 2/3$. Find the exact value of $\sin(\alpha - \pi/4)$.

Solution: $\sin(\alpha - \pi/4) = \sin\alpha\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\alpha$, since α is in second region we find $\cos\alpha = -\frac{\sqrt{5}}{3}$ by the right triangle, so the answer is $\frac{2\sqrt{2}+\sqrt{10}}{6}$. **Q.E.D.**

(9)
What are the domain and the range of $\sin^{-1}$?

Solution: Domain is $[-1, 1]$, range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. **Q.E.D.**

(10)
Find the exact value of $\tan^{-1}(1)$.

Solution: $\tan\frac{\pi}{4} = 1$ and $\frac{\pi}{4}$ is in the range of our function so answer is $\frac{\pi}{4}$. **Q.E.D.**

(11)
Find all real numbers θ solving the trigonometric equation $\cos 2\theta = 1$.

Solution: $\cos 2\theta = 1 = \cos 0$, so $2\theta = 0 + 2k\pi$. So answer is $\theta = k\pi$ where k are integers. **Q.E.D.**

(12)
Solve the following trigonometric equation in the interval $0 \leq \theta < 2\pi$

$$\cos^2 + 2\cos\theta - 3 = 0$$

Solution: $x = \cos\theta$, so equation is now $x^2 + 2x - 3 = 0$ which has roots 1 and -3. $-3 = x = \cos\theta$ has no solution, $1 = x = \cos\theta$ has 1 solution in that interval which is $\theta = 0$. So the only answer is $\theta = 0$. **Q.E.D.**

(13)
A triangle has one side of length 4 and the two angles adjacent to this side measure 50^0 and 70^0 respectively. Solve the triangle and find its area.

Solution: This is simple Sine Law, $\frac{4}{\sin 60^o} = \frac{b}{\sin 50^o} = \frac{c}{\sin 70^o}$. With this you can find a, b and then using Heron's formula you can find the area, here I need a calculator which right now I don't have one.**Q.E.D.**

(14)
Find one of the polar coordinates of the point P whose rectangular coordinates are $(x, y) = (1, \sqrt{3})$.

Solution: By the simple formula we can find $r = \sqrt{1^2 + 3} = 2$ and using the corresponding equations we see that $2, \frac{\pi}{3}$ satisfies our point.**Q.E.D.**

(15)
Write the polar equation

$$r = \frac{3}{r - 2\sin\theta}$$

in rectangular coordinates.

Solution:$r = \frac{3}{r - 2\sin\theta}$ is equal to $r^2 - 2r\sin\theta = 3$ which is equivalent to $r^2 - 2y = 3$. So we get here $r^2 = 3 + 2y$ which is equal to $x^2 + y^2 = 3 + 2y$. So we get $x^2 + y^2 - 2y + 1 = 3 + 1$ which is equal to $x^2 + (y-1)^2 = 4$ which as you can see a circle with radius 2 and center $(0, 1)$. **Q.E.D.**

(16)
Write in standard form the complex number $\frac{2+i}{3i}$

Solution: $\frac{2+i}{3i} = i(\frac{2+i}{-3}) = \frac{2i-1}{-3} = \frac{1-2i}{3}$. **Q.E.D.**

(17)
Let z be a complex number written in polar form as $z = 3(\cos 20^0 + i \sin 20^0)$. Write in standard form the number z^9.

Solution: By De Moivre's theorem $z^9 = 3^9(\cos(9 * 20^0) + i \sin(9 * 20^0))$ which is equal to $3^9(\cos 180^o + i \sin 180^o) = -3^9$ since $\cos 180^o = -1$ and $\sin 180^o = 0$. **Q.E.D.**

(18)
Find all the three roots of the cubic equation

$$x^3 + x^2 + 3x - 5 = 0$$

Solution: By Rational Root Theorem we see that 1 is a root. Dividing this polynomial to $x - 1$ through syntethic division we see that the other factor is $x^2 + 2x + 5$, so the other two roots are the roots of this quadratic polynomial, which are $\frac{-2+\sqrt{-16}}{2}$ and $\frac{-2-\sqrt{-16}}{2}$, i.e. $-1+2i$ and $-1-2i$. **Q.E.D.**

(19)
Find the complex roots of

$$z^3 = 1$$

and describe their position in the complex plane.

Solution: Writing $z = r(\cos\theta + i \sin\theta)$ we see that equation is equal to $r^3(\cos 3\theta + i \sin 3\theta) = 1(\cos 0 + i \sin 0)$ since $1 = 1(\cos 0 + i \sin 0)$ in the complex plane. So solving this equation gives us the solutions $z_k = 1(\cos(\frac{0+2k\pi}{3}) + i \sin(\frac{0+2k\pi}{3}))$ where $k = 0, 1, 2$. On the complex plane these points correspond to the vertices of an equilateral triangle on the unit circle, it is easy to see where these vertices are located. **Q.E.D.**

(20)
Establish if the following system of linear equations is consistent or not. If it

is consistent find all its solutions.

$$\begin{cases} x + y - 3z &= 1 \\ 2x - 4y + z &= 5 \\ x - z &= 4 \end{cases}$$

Solution: From third equation we see that $x = 4+z$, if we replace that in the second and first equation we get $3z - 4y = -3$ from second, $y - 2z = -3$ from first. So left sides of these equations are equal since they both are equal to -3, so $3z - 4y = y - 2z$, so $y = z$. Putting this into first equation we get, $4 - z = 1$, $z = 3$. So $(7, 3, 3)$ is the solution. **Q.E.D.**