MATH 1272 MIDTERM EXAM III,NOVEMBER 29,FALL 2001

Name ______________________ Discussion Section ______________
ID ______________________

(1) **(15 points)** Find the sum of the geometric series:

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{3^n}.$$

Solution: Using our famous geometric sum formula:

$$\frac{3}{4} = \frac{1}{1+\frac{1}{3}} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{3^n}.$$

So answer is $\frac{3}{4}$. **Q.E.D.**

(2) **(20 points)** Determine whether the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$$

is convergent or divergent. Hint: Try to see how nth term $a_n = \frac{\sqrt{n+1}-\sqrt{n}}{n}$ behaves as $n \to \infty$ then see if you can use comparison or integral test or combination of the both.

Solution: We need to play with the equation a little bit:

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n} = \sum_{n=1}^{\infty} \left(\frac{(\sqrt{n+1}-\sqrt{n})}{n}\frac{(\sqrt{n+1}+\sqrt{n})}{(\sqrt{n+1}+\sqrt{n})}\right) = \sum_{n=1}^{\infty} \frac{1}{n(\sqrt{n+1}+\sqrt{n})}.$$

Also:

$$\sum_{n=1}^{\infty} \frac{1}{n(\sqrt{n+1}+\sqrt{n})} \le \sum_{n=1}^{\infty} \frac{1}{2n\sqrt{n}}.$$

So by p-test the latter series is convergent since $p = \frac{3}{2} > 1$. That means our series is convergent by comparison test. **Q.E.D.**

(3) **(20 points)** How many terms of the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{5+\log_2 n}$$

do we need to add in order to find the sum with error ≤ 0.1?

Solution: $|R_n| \leq b_{n+1}$, so $|R_n| \leq b_{n+1} \leq 0.1$. So $\frac{1}{5+\log_2 n+1} \leq 0.1$ which gives $10 \leq 5+\log_2 n+1$.So $32 \leq n+1$ which tells us that any n bigger than or equal to 31 would work. **Q.E.D.**

(4) **(20 points)** Find the radius R of convergence of the power series

$$\sum_{n=0}^{\infty} \ln(n+1)\, x^n.$$

Is it convergent for $x = \pm R$?

Solution: Using the Ratio Test we see that $\lim_{n\to\infty} |\frac{\ln(n+2)x}{\ln(n+1)}| = |x|$ since $\lim_{n\to\infty} \frac{\ln(n+2)}{\ln(n+1)} = 1$ bye L'Hospital's Rule. So $|x| < 1$ is the region that this series converges. So $R = 1$. Now for $x = 1$ series is equal to:

$$\sum_{n=0}^{\infty} \ln(n+1)\,.$$

which is divergent since $\lim_{n\to\infty} \ln(n+1) = \infty$. For $x = -1$ series is equal to:

$$\sum_{n=0}^{\infty} \ln(n+1)\,(-1)^n$$

which is divergent since $\lim_{n\to\infty}(-1)^n \ln(n+1) = \pm\infty$. **Q.E.D.**

(5) **a) (10 points)** Find the Taylor polynomial $T_2(x)$ of degree $n = 2$ about the point $a = 1$ for the function $f(x) = x \ln x$.

b)(15 points) Use Taylor's inequality to indicate a bound for the error $R_2(x)$ of approximation in the interval $|x-1| \leq \frac{1}{2}$.

Solution: a) We need the derivatives first:$f^{(0)}(1) = 0, f^{(1)}(1) = \ln 1+1 = 1, f^{(2)}(1) = \frac{1}{1} + 0 = 1$. So our polynomial is going to be $T_2(x) = 0 + 1 * (x -$

$1) + 1 * (x-1)^2 = x^2 - x = x(x-1)$. **Q.E.D.**

Solution: b) $|R_2(x)| \leq \frac{M}{3!}(x-1)^3$ where $|f^{(3)}(x)| \leq M$ in our interval $|x-1| \leq \frac{1}{2}$. Since $f^{(3)}(x) = -\frac{1}{x^2}$ we see that M takes its best at $x = \frac{1}{2}$. So $M = 4$ is the least possible number for us. Since $|x-1| \leq \frac{1}{2}$ we have the bound as $B = \frac{1}{12}$. **Q.E.D.**