

1. Evaluate $\iint_S (x+z)dS$ where S is the portion of the cylinder $y^2 + z^2 = 9$ with $0 \leq x \leq 4$, $y \geq 0$, $z \geq 0$.

Answer: $12\pi + 36$ [use $\vec{\Phi}(u, v) = (u, \cos v, \sin v)$]

2. Evaluate $\iint_S \sqrt{x^2 + y^2} dS$ where S is the portion of the cone $z = 4 - \sqrt{x^2 + y^2}$ with $0 \leq z \leq 4$.

Answer: $\frac{128\sqrt{2}\pi}{3}$ [use $\vec{\Phi}(r, \theta) = (r \cos \theta, r \sin \theta, 4 - r)$]

3. Evaluate $\iint_S (x, y, z) \cdot d\vec{S}$ where S is the portion of the paraboloid $z = 4 - x^2 - y^2$ above the xy -plane and a normal vector is pointing upward.

Answer: 24π [use $\vec{\Phi}(r, \theta) = (r \cos \theta, r \sin \theta, 4 - r^2)$]

4. Evaluate $\iint_S (3z\vec{i} - 4\vec{j} + y\vec{k}) \cdot d\vec{S}$ where S is the portion of the plane $x + y + z = 1$ in the first octant and S has an upward pointing normal.

Answer: $-\frac{4}{3}$ [use $\vec{\Phi}(u, v) = (u, v, 1 - u - v)$]

5. Evaluate $\int_C \vec{F} \cdot d\vec{s}$ where $\vec{F}(x, y, z) = (-y^2, z, x)$ and C is the triangle with vertices $(3, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$.

Answer: -9 [convert to surface integral of $\nabla \times \vec{F}$ using Stokes' Theorem and use $\vec{\Phi}(u, v) = (u, v, 6 - 2u - 2v)$]

6. Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ where S is the portion of the sphere $x^2 + y^2 + z^2 = 4$ above the cone $x^2 + y^2 = z^2$ and $\vec{F}(x, y, z) = (x^2, xz^2, xyz)$.

Answer: 4π [convert to line integral of \vec{F} using Stokes' Theorem and use $\vec{c}(\theta) = (\sqrt{2} \cos \theta, \sqrt{2} \sin \theta, \sqrt{2})$]

7. Evaluate

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 x dz dy dx$$

Answer: 0 [change coordinates to cylindrical coordinates. The new integral should be:

$$\int_0^4 \int_0^{2\pi} \int_0^{\sqrt{z}} r^2 \cos \theta dr d\theta dz$$

]

8. Evaluate $\iiint_Q z dV$ where Q is the portion of the ball $x^2 + y^2 + z^2 \leq 9$ above the cone $x^2 + y^2 = z^2$.

Answer: $\frac{81\pi}{8}$ [use spherical coordinates, with $0 \leq \theta \leq 2\pi$, $0 \leq \phi \leq \frac{\pi}{4}$, $0 \leq \rho \leq 3$]

9. Evaluate $\iint_R (x+y)^2 \sin^2(x-y) dA$ where R is the rectangle with vertices $(0, 1), (1, 2), (2, 1),$ and $(1, 0)$. (Hint: let $u = x + y$, $v = x - y$ and change coordinates.)

Answer: $\frac{13}{6}(2 - \sin 2)$ [remember to use the Jacobian determinant]

10. Find an equation of the tangent plane to $\vec{\Phi}(u, v) = u\vec{i} + v\vec{j} + (u^2 + v^2)\vec{k}$ at the point $(1, 2, 5)$.

Answer: $2x - 4y - z = -11$

11. Evaluate $\int_C e^x \sin y dx + e^x \cos y dy$ where C is the curve parameterized by $\vec{c}(t) = (t - \sin t, 1 - \cos t)$ for $\frac{\pi}{2} \leq t \leq \pi$.

Answer: $e^\pi \sin 2 - e^{(\pi/2-1)} \sin 1$ [Note that the vector field in question is conservative, so you can find a potential function and use the fundamental theorem of line integrals]

12. Let $D^* = [0, 1] \times [0, 2]$.

(a) Show that $T_1(x^*, y^*) = (2x^* + y^*, x^* - 3y^*)$ is one-to-one on D^* .

Answer: Suppose $T_1(x_1, y_1) = T_1(x_2, y_2)$. Then, we get that $(2x_1 + y_1, x_1 - 3y_1) = (2x_2 + y_2, x_2 - 3y_2)$. So, $2x_1 + y_1 = 2x_2 + y_2$ and $x_1 - 3y_1 = x_2 - 3y_2$. Multiplying both sides of $2x_1 + y_1 = 2x_2 + y_2$ by 3 and adding the result to $x_1 - 3y_1 = x_2 - 3y_2$ gives

$7x_1 = 7x_2$. Thus, $x_1 = x_2$. So, the equation $2x_1 + y_1 = 2x_2 + y_2$ becomes $y_1 = y_2$. Thus, if $T_1(x_1, y_1) = T_1(x_2, y_2)$, then $(x_1, y_1) = (x_2, y_2)$. So, T_1 is one-to-one on D^* .

(b) Show that $T_2(x^*, y^*) = ((x^*)^2 - (y^*)^2, x^* - y^*)$ is NOT one-to-one on D^* .

Answer: $T_2(0, 0) = (0, 0) = T_2(1, 1)$. So, T_2 is not one-to-one.

13. Describe the surface:

(a) $\vec{\Phi}(u, v) = (3 \cos u \sin v, 3 \sin u \sin v, 3 \cos v)$ for $0 \leq u \leq 2\pi$, $0 \leq v \leq \pi$

Answer: sphere of radius 3

(b) $\vec{\Phi}(u, v) = (u \cos v \sin \frac{\pi}{8}, u \sin v \sin \frac{\pi}{8}, u \cos \frac{\pi}{8})$ for $0 \leq u \leq 3$, $0 \leq v \leq 2\pi$

Answer: cone opening upward

(c) $\vec{\Phi}(u, v) = (u \cos \frac{\pi}{8} \sin v, u \sin \frac{\pi}{8} \sin v, u \cos v)$ for $0 \leq u \leq 3$, $0 \leq v \leq \pi$

Answer: half disk