PDE seminar
University of Minnesota
Wednesday, October 31, 2012

Speaker: **Nathan Totz (Duke University)**
Title: **A rigorous justification of the modulation approximation to the 2D full water wave problem.**

Abstract: In this joint work with Sijue Wu (U. Mich.), we consider solutions to the 2D inviscid infinite depth water wave problem neglecting surface tension which are to leading order wave packets with small amplitude and slow spatial decay that are balanced. Multiscale calculations to suppress secular growth on $O(\epsilon^{-2})$ time scales formally suggest that such solutions have modulations B that evolve on those time scales according to a focusing cubic NLS equation. Justifying this rigorously is a real problem, since standard existence results do not yield solutions which exist for long enough to see the NLS dynamics. Nonetheless, given initial data within $O(\epsilon^{3/2})$ of such a wave packet in L^2 Sobolev space, we show that there exists a unique solution to the water wave problem which remains within $O(\epsilon^{3/2})$ to the formal approximation for times of order $O(\epsilon^{-2})$. This is done by using a formulation of the evolution equations for the water wave problem developed by Sijue Wu with no quadratic nonlinearity. If time permits, we will also discuss work in the analogous problem in the 3D setting.