PDE seminar
University of Minnesota
Wednesday, September 19

Speaker: **Svitlana Mayboroda**
Title: **Well-posedness in L^p for elliptic boundary value problems.**

Abstract: One of the simplest and the most important results in elliptic theory is the maximum principle. It provides sharp estimates for the solutions to elliptic PDEs in L^∞ in terms of the corresponding norm of the boundary data. It holds on arbitrary domains for all (real) second order divergence form elliptic operators $-divA\nabla$. The well-posedness of boundary problems in L^p, $p < \infty$, is a far more intricate and challenging question, even in a half-space, $\mathbb{R}^{n+1}_+$. In particular, it is known that some smoothness of A in t, the transversal direction to the boundary, is needed.

In the present talk we shall discuss the well-posedness in L^p for elliptic PDEs associated to matrices A independent on the transversal direction to the boundary. In combination with our earlier perturbation theorems, this result shows that the Dirichlet boundary value problem is well-posed in some L^p, $p < \infty$, whenever (roughly speaking) $|A(x,t) - A(x,0)|^2 dxdt/t$ is a small Carleson measure. Such a result was only known in the setting of real symmetric matrices [D. Jerison, C. Kenig, 1981]. The non-symmetric case was open since then, and ultimately had to be approached by completely different techniques. In 2000 Kenig, Koch, Pipher, and Toro established the well-posedness for non-symmetric matrices in dimension 2. The present work pertains to all dimensions $n \geq 2$.

This is joint work with S. Hofmann, C. Kenig, and J. Pipher.