High-Order Discontinuous Galerkin Methods for Fluid and Solid Mechanics

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Abstract: Discontinuous Galerkin (DG) methods have gained popularity because of their ability to discretize conservation laws with high-order accuracy on complex geometries. They have the potential to produce highly accurate solutions with minimum numerical dissipation, which makes them attractive for aerodynamic applications such as aeroacoustics and turbulence simulations, and other problems involving wave propagation, multiple scale phenomena, and non-linear interactions. Moreover, DG methods can be used with unstructured meshes of tetrahedra, which appears to be a requirement for real-world geometries.

However, as of today DG methods have only been demonstrated in the research community for academic applications with relatively simple physics and geometries. One of the main challenges is to extend these methods to relevant engineering applications. This requires the ability to generate appropriately stretched high-order meshes and robust discretizations, maintain stability in the presence of shocks, and solve the resulting systems of equations in a competitive manner. We present a number of new developments in our work on DG methods: (i) The Compact Discontinuous Galerkin (CDG) method, an accurate and low cost discretization for viscous terms. (ii) A Newton-GMRES preconditioner based on p-multigrid and block-ILU smoothing, with optimized element numbering using a Minimum Discarded Fill method. (iii) A stabilization technique based on artificial viscosity which gives subgrid accuracy for shock capturing and the ability to handle RANS problems. (iv) A high-order ALE formulation for deforming geometries based on mappings. We show examples of aeroacoustic simulations, high-Reynolds

number transonic flows, non-linear solid dynamics, and applications involving fluid-structure interaction with particular emphasis on flapping flight.