1. The cosine function, $\cos x$, is given by the following power series: $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$. This power series has a radius of convergence $R$ of $+\infty$. Find the power series for $\sin^2 x$, i.e. a power series $\sum_{n=0}^{\infty} \alpha_n x^n$ whose radius of convergence is $+\infty$ and such that $\sin^2 x = \sum_{n=0}^{\infty} \alpha_n x^n$ for every real number x. (Hint: use trigonometric identities.)

2. Let $\sum_{n=0}^{\infty} \alpha_n x^n$ be a power series with all $\alpha_n > 0$. Assume the sequence $\frac{\alpha_n}{\alpha_{n+1}}$ converges to a number $r (\geq 0)$. Show $r$ is the radius of convergence of the series.

3. The exponential function $e^x$ is given by the power series $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$. Find the power series for $\frac{d}{dx}(\frac{e^x - 1}{x})$. Show that $1 = \sum_{n=1}^{\infty} \frac{n}{(n+1)!}$.