RESEARCH STATEMENT

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Symplectic Topology is the study of smooth manifolds X which admit a closed non-degenerate 2-form ω, called a symplectic form. Symplectic manifolds are interesting for many reasons, among them is their natural appearance in physics (for example in classical mechanics and string theory) and their similarity/differences to complex manifolds (All symplectic manifolds admit almost complex structures.). The two basic questions asked in symplectic topology are the following:

(1) Which smooth manifolds X admit symplectic structures?
(2) If X admits a symplectic structure, which classes $\alpha \in H^2(X)$ are representable by orientation compatible symplectic forms? The set of such classes is called the symplectic cone $\mathcal{C}_X$.

The work detailed in Sections 1 and 2 is directly related to the latter question. The search for invariants of symplectic manifolds has been very fruitful in providing partial answers to these and other questions as well as illuminating relations between symplectic topology and other fields.

1. Relative Methods in Symplectic Topology

Many surgery constructions on symplectic manifolds have been defined in order to construct symplectic manifolds with certain properties. Most involve surgery along submanifolds. Therefore, understanding symplectic structures with respect to fixed submanifolds has become a central issue in researching symplectic manifolds. We are particularly interested in codimension 2 symplectic submanifolds due to the symplectic cut/sum constructions ([Gom95], [MW94], [Ler95]). This operation is performed on two symplectic manifolds X and Y, each containing a symplectic codimension 2 submanifold V, by removing normal neighborhoods of V in each and gluing the boundaries together in an orientation compatible form to produce a manifold $X\#_V Y$. This operation is originally defined in the smooth category, it was shown in [Gom95] that this can be done in the symplectic category as well.

1.1. **Relative Symplectic Cone.** We define the relative symplectic cone $\mathcal{C}_X^V$ as the set of classes $\alpha \in H^2(X)$ which are representable by a orientation compatible symplectic form making a fixed codimension 2 submanifold V symplectic ([DL08a]). This is the natural set of symplectic structures to consider when performing the symplectic sum operation.

Date: July 12, 2009.

We use the relative cone in [DL08a] to determine the symplectic cone of certain manifolds: The relative cones of X and Y combine nicely along forms which are identical on V to produce a subset of the symplectic cone of $X\#_V Y$. We show that under certain topological restrictions, this subset is the whole cone. The topological restriction explicitly prevents the existence of classes in $X\#_V Y$ which degenerate trivially under $X\#_V Y \to X \sqcup_V Y$. Removing or understanding the limits of this restriction completely would be a nice result and allow for a determination of the symplectic cone of the elliptic manifolds $E(2n)$.

We completely determine the relative cone for manifolds with $b^+ = 1$ and Geiges has determined the relative cone with respect to the fiber torus of T^2 fibrations over T^2 ([Gei92]). We plan to extend these calculations to K3 surfaces.

It is expected, that the relative cone also provides the natural set of symplectic classes associated to relative Gromov-Witten theory ([IP03], [LR01], [LiJ02]).

1.2. **Relative Ruan and Gromov-Taubes Invariants.** Gromov's paper [Gro85] initiated the intense study of pseudoholomorphic curves in symplectic manifolds as a means to construct symplectic invariants. This has led to a wide range of invariants for symplectic manifolds which "count" such curves. After the development of the symplectic sum and symplectic cut ([Ler95], [Gom95], [MW94]), research turned also to developing invariants relative to a fixed symplectic submanifold (or a divisor) V of codimension 2. A variety of relative invariants have been developed which consider curves in X which contact V in a specified manner ([IP03], [LR01], [LiJ04]).

A more natural object of study in the symplectic category are embedded symplectic submanifolds. This topic was initially studied by Ruan [Rua94], who initiated the development of invariants which count embedded curves in four-manifolds. Moreover, in a series of fundamental papers ([Tau00a], [Tau00b]), Taubes defined invariants, which give a delicate count of embedded, possibly disconnected, curves (This count refines Ruan's earlier results, in particular providing a detailed analysis of the behavior of square 0- tori and their multiple covers.), and equated them with Seiberg-Witten invariants. We will refer to these invariants as Gromov-Taubes invariants.

1.2.1. *Relative Ruan Invariants.* In [DL08b], we define relative Ruan invariants for symplectic four-manifolds, which count embedded connected symplectic curves which contact a fixed symplectic submanifold V at prescribed points with prescribed contact orders (in addition to insertions on $X\backslash V$). These invariants are deformation invariants of the triple (X, V, ω), note however that due to the relative setting, the symplectic structures must all make the submanifold V symplectic. This connects the rel. Ruan invariants to the relative symplectic cone defined in [DL08a]. The construction of these invariants generally follows Ruan and Taubes' work, however with one crucial difference: We must preserve the symplecticity of the submanifold V, which means our calculations must be restricted to allow only almost complex structures J on X which keep V pseudoholomorphic. This restriction needs careful consideration when attempting to do surjectivity arguments, in particular as the submanifold V can itself appear as a component. This forms the core of [DL08b].

The subtleties present when curves limit into V become apparent in the determination of the Kodaira dimension of a rational blow-down, see 3.1. In particular, if V is non-generic, then curves with V components may lie in strata which have the expected dimension or higher. Hence such curves cannot necessarily be avoided through a generic choice of data, but will need a virtual neighborhood method to define meaningful invariants.

Moreover, as in [IP03], the invariants we define can be refined to include rim tori decompositions. These refined invariants are expected to be isotopy invariants of the triple (X, V, ω). It should be possible for these invariants to detect different isotopy classes of the class of the submanifold V. The study of non-isotopic submanifolds is very active, see the following for more references: [PPV07], [FS99], [Vid05], [Vid04], and [EP04].

1.2.2. *Relative Gromov-Taubes Invariants.* The rel. Ruan invariant gives a count of connected curves which are not tori with trivial normal bundle or covers thereof. Taubes' invariant giving a count of such tori can be used in conjunction with the rel. Ruan invariant to define a relative Gromov-Taubes invariant counting relative possibly disconnected curves in (X, V). This is the relative version of the Gromov-Taubes invariant defined in [Tau00a].

1.2.3. *General Program.* These invariants are the first step in an extended program. The goal is to develop a sum formula for Gromov-Taubes and relative Gromov-Taubes invariants, similar as was done in Gromov-Witten theory, and connect the relative Gromov-Taubes invariants to relative Seiberg-Witten invariants.

A sum formula should make computation of the Gromov-Taubes invariant easier, as has happened in Gromov-Witten theory. Furthermore, using the degenration formula, we hope to construct absolute - relative relations as developed in [MP08] and [HLR08] for Gromov-Witten invariants. Understanding this relationship for embedded invariants and determining whether they are also determined by the topology of the underlying pair (X, V) should be interesting.

1.2.4. *Further Work.* There exist a number of invariants in the relative setting, for example relative Seiberg-Witten, relative Oszvath-Szabo and general relative Gromov-Witten invariants. Each highlights a different aspect of the underlying symplectic manifold (or of the pair (X, V)). A detailed comparison of these invariants should provide interesting insight into symplectic structures and make computation of these various invariants easier. Furthermore, it would be interesting to understand the relation between embedded invariants, as we have constructed, and invariants of immersed curves (general Gromov-Witten invariants, not necessarily relative). For work on relative theories and product formulas, see [Tau01], [Par02], [MMS97], [MST96], as well as [LiJ02], [LR01] and [IP04].

Explicit calculations of relative Ruan and Gromov-Taubes invariants for a range of examples would be useful in understanding the issues above. In order to build up a collection of such examples, we would like to determine these invariants for K3 surfaces, building on work in [MP08] and [DL08b]. This involves proving a number of vanishing results for relative embedded surfaces. Moreover, calculating the invariants of $\mathbb{P}^1$-bundles would be a necessary first step towards an absolute-relative correspondence.

2. Symplectic Cones

The symplectic cone $\mathcal{C}_X$ is the set of classes $\alpha \in H^2(X)$ which are representable by an orientation compatible symplectic form. Determining the symplectic cone is a challenging task, it is known for few manifolds: In dimension 4, the symplectic cone has been determined in the following cases:

- S^2-bundles ([McD94]),
- symplectic T^2-bundles over T^2 ([Gei92]),
- all $b^+ = 1$ manifolds ([LL01], see also [McD94], [Bir97]),
- minimal manifolds underlying a Kähler surface with Kodaira dimension 0 ([TJL08]) (A smooth 4-manifold M is said to be minimal if it contains no exceptional class, i.e. a degree 2 homology class represented by a smoothly embedded sphere of self intersection -1.), and
- Friedl and Vidussi (see [FV08a] and [FV08b]) determined the symplectic cone of a product S^1- bundle over any 3-manifold or a S^1-bundle over a graph manifold in terms of the Thurston norm ball of the 3-manifold.

We extend the set of manifolds for which the symplectic cone is known to certain T^2-fibrations in [DL08a], including examples with symplectic Kodaira dimension 2 and $b^+ > 1$, using the method described in the previous Section. The results of Friedl-Vidussi overlap ours for the product T^2-fibrations $T^2 \times \Sigma_g$.

We are particularly interested in computing the symplectic cone of elliptic surfaces $E(n)$. So far, we have encountered considerable difficulties in doing so. An understanding of how the symplectic cone of the elliptic surface $E(1)$ leads to the symplectic cone of an elliptic K3 surface may provide insight into how to construct the symplectic cone of elliptic surfaces $E(n)$, $n > 2$. The deeper issue lies hidden in the generation of new classes in the homology of $E(n)$ which did not exist in $E(n-1)$ and were not generated from nontrivial classes under the symplectic sum operation. This can be nicely seen in attempts at calculating the symplectic cone of $K3\#_{T^2}K3$. In either case, a better understanding of relative structures is needed.

3. Kodaira Dimension of 4-Manifolds

Kodaira dimension is defined for compact complex manifolds in terms of the plurigenera of the manifold ([BHPV04]). On a symplectic manifold, the Kodaira dimension is defined in terms of the canonical class and the symplectic form ([MS96], [TJL06a], [LeB96]). These two definitions are equivalent on manifolds admitting both complex and symplectic structures ([DZ08]).

3.1. **and Fiber Sums.** The symplectic fiber sum constructs a symplectic manifold $M + X\#_V Y$, can we determine the Kodaira dimension $\kappa(M)$? To answer this question, we must first determine how minimality changes under the fiber sum operation, as Kodaira dimension is defined on the minimal model of M. For fiber sums along surfaces with positive genus, this question has been answered in [Ush06]. The behavior of Kodaira dimension for such sums was then described in [LY07].

In the genus 0 case, this behavior is described in [D09]. To determine minimality, we use the sum formula for GW-invariants as defined in [LR01]. However, in this case we must take care to account for curves with components mapping to the fixed submanifold V. This requires some delicate argumentation. We obtain the following

Theorem 3.1 ($g(V) > 0$: [Ush06], $g(V) = 0$: [D09])**.** *Let M be the fiber sum $X\#_V Y$ along an embedded surface V.*

1. *The manifold M is not minimal if*
   - *$X \backslash V$ or $Y \backslash V$ contains an embedded symplectic sphere of self-intersection -1 or*
   - *$Y = \mathbb{C}P^2$, V is an embedded 4-sphere in class $\mathfrak{V} = 2\mathfrak{H} \in H_2(\mathbb{C}P^2, \mathbb{Z})$ and X has at least 2 exceptional curves E_i each meeting the submanifold $V \subset X$ in a single point with multiplicity 1, i.e. $E_i \cdot \mathfrak{V} = 1$.*
2. *If Y is a S^2-bundle over a genus g surface and V is a section of this bundle then M is minimal if and only if X is minimal.*
3. *In all other cases M is minimal.*

Subsequently we prove the following theorem, the higher genus proof can be found in [LY07].

Theorem 3.2. *Let $M = X\#_V Y$ be a symplectic fiber sum along an embedded symplectic surface V. Then the Kodaira dimension is non-decreasing, i.e.*

$$\kappa(M) \geq \max\{\kappa(X), \kappa(Y), \kappa(V)\}.$$

In [Ush09], it was shown that fiber sums along symplectic submanifolds with positive genus do not produce any new diffeomorphism classes of symplectic manifolds having Kodaira dimension 0, which have been classified up to homology type in [TJL06a] and [TJL06b]. In [D09] we show that this also holds true in the genus 0 case. More precisely, we show that the only manifold with $\kappa(M) = 0$ which can be obtained from a fiber sum along a sphere is the Enriques surface.

Of particular interest in the above calculations is the rational blow-down of a -4-sphere. A -4-sphere is non-generic in the sense of Taubes and thus carries an obstruction bundle. This can produce curves which have V components and lie in strata of the appropriate moduli space which have dimension up to 2 larger than the expected dimension. As we need to consider disconnected curves in determining the presence of an exceptional sphere, such higher level curves need to be carefully ruled out. This is a delicate procedure and clearly illustrates the issues involved when V is non-generic.

The rational blow-down of a -4-sphere is the simplest case of a more general rational blow-down procedure defined in [FS97] and shown to be possible in the symplectic category in [Sym98]. The next step in the above process is to extend the results obatined here for the -4-sphere to this more general procedure. This will involve completely different methods, as the general procedure is no longer a fiber sum.

3.2. **and Lefschetz Fibrations.** In [DZ08] we extend the definition of Kodaira dimension to Lefschetz fibrations and pencils. This allows for a purely combinatorial description of the dimension. Attempts to show that these definitions are all equivalent have led to interesting questions concerning symplectic manifolds of Kodaira dimension > 0 as well as the structure of Lefschetz fibrations with tori as fiber or base. Of particular interest are the following two questions:

(1) If the total space of the Lefschetz fibration admits a complex structure, can we find one compatible with the fibration? If not, how much must we perturb the fibration to obtain a holomorphic fibration?
(2) Does the slope inequality of Xiao ([Xia87]) hold for symplectic fibrations?(See also [AB00])

In [TJL08] it has been shown, that there exist Kähler manifolds admitting Lefschetz fibrations over S^2 which are not holomorphic. Whether these can be perturbed to become holomorphic is open. There exist perturbation results which may provide a framework for answering these questions, see for example [JY93].

Some results are known with respect to (2): For hyperelliptic and holomorphic fibrations the answer is positive ([End00], [Mat86]). Otherwise this appears open. It is intimately related to (1): If we understand the answer to that question, then we can apply Xiao's results to the perturbed fibrations. So long as the perturbations do not change the slope of the fibration, we can use this to work towards an answer to (2).

It has recently been shown, that every smooth four manifold admits a broken Lefschetz fibration ([AK08], [Lek07]). Therefore, an extension of our results to such fibrations would provide a definition of Kodaira dimension for smooth four-manifolds. Moreover, work by Lebrun ([LeB96], [LeB99], [LeB97]) has shown an intimate connection between Yamabe invariants and complex Kodaira dimension. The Yamabe Invariant would be a possible definition of Kodaira dimension on smooth four manifolds, albeit not necessarily a natural or good one. A first step towards a better understanding of the underlying structures would be to extend the relations found by LeBrun to the symplectic setting.

For compact complex manifolds as well as, to a somewhat lesser degree, for symplectic manifolds, the classification induced by Kodaira dimension has been extremely useful in understanding four-manifolds. A completion of the classification of symplectic manifolds of Kodaira dimension 0 (see [TJL06a]) as well as a better understanding of manifolds in dimension 1 is needed. Furthermore, one can hope to find a classification scheme on symplectic (or maybe even smooth) four-manifolds in the style of the Enriques-Kodaira classification on compact complex manifolds.

In particular, a deeper understanding of these different invariants and definitions might lead to a natural definition of Kodaira dimension on smooth four-manifolds as well as provide insight as to an extension of the non-complex definitions of Kodaira dimension to higher dimensions. So far, too little is known of the properties of symplectic four-manifolds in Kodaira dimension 1 and 2.

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