1. For the following regions R, compute the specified regions.

   (a) R is the rectangle with vertices $(-2,2),(0,2),(0,0),(-2,0)$

$$\iint_R (3x^2 + 2y^3) dA$$

We would usually draw a picture of the region here, but this region is very simple, so there is no need. We can see from the points that x varies freely between -2 and 0, while y varies freely between 0 and 2. This gives the setup for the integral

$$\int_0^2 \int_{-2}^0 \left(3x^2 + 2y^3\right) dxdy$$

Which we can compute

$$\int_0^2 \int_{-2}^0 \left(3x^2 + 2y^3\right) dxdy = \int_0^2 \left(x^3 + 2y^3 x \Big|_{-2}^{0} \right) dydx =$$

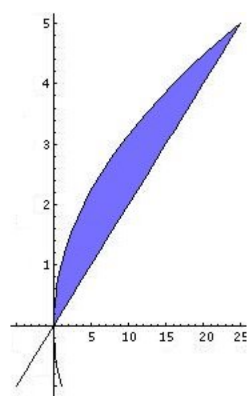
$$= \int_0^2 \left(8 + 4y^3\right) dydx =$$

$$= 8y + y^4 \Big|_0^2 = 32$$

   (b) R is the region bounded by $x = y^2$ and $x = 5y$

$$\iint_R ye^x dA$$

First we setup the bound on this integral. The easiest way to do this is to draw a picture



We see that the bounds on y vary between between $y = 0$ and $y = 5$. Further we see that for any y, x can vary between y^2 and $5x$. This gives us all the information we need to setup the integral.

$$\int_0^5 \int_{y^2}^{5y} ye^x dxdy$$

Now we can compute this integral

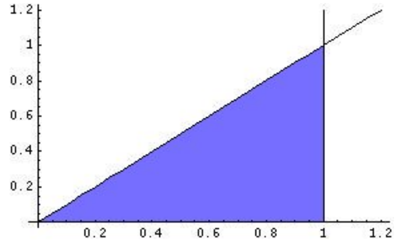
$$\begin{aligned}
\int_0^5 \int_{y^2}^{5y} ye^x dxdy &= \int_0^5 ye^x \Big|_{y^2}^{5y} = \\
&= \int_0^5 ye^{5y} - ye^{y^2} = \\
&= \left(\frac{ye^{5y}}{5} - \frac{e^{5y}}{25} - \frac{e^{y^2}}{2} \right) \Big|_0^5 = \\
&= \frac{27 + 23e^{25}}{50}
\end{aligned}$$

2. Consider the integral: $\int_0^1 \int_y^1 x^2 dxdy$

   (i) Compute the Integral.
   Computing this integral as given is straightforward.

$$\begin{aligned}
\int_0^1 \int_y^1 x^2 dxdy &= \int_0^1 \frac{1}{3} x^3 \Big|_y^1 dy \\
&= \int_0^1 \frac{1}{3} - \frac{1}{3} y^3 dy \\
&= \frac{1}{3} y - \frac{1}{12} y^4 \Big|_0^1 \\
&= \frac{1}{4}
\end{aligned}$$

   (ii) Sketch the region you are integrating over.



(iii) Change the order of integration.

Reversing the order of integration we look now want to setup the integral is taken $dydx$. So we look along the x axis and see that our integral will still go between 0 and 1. Then we look along the y axis, and for any $x \in (0, 1)$ we know $0 \leq y \leq x$. This gives us all the information we need to setup this integral.

$$\int_0^1 \int_0^x x^2 dydx$$

(iv) Compute the new integral.

This computation is even easier.

$$\begin{aligned}
\int_0^1 \int_0^x x^2 dydx &= \int_0^1 x^2 y \Big|_0^x \\
&= \int_0^1 x^3 \\
&= \frac{1}{4} x^4 \Big|_0^1 \\
&= \frac{1}{4}
\end{aligned}$$

The fact that the answers agree should come as no surprise.

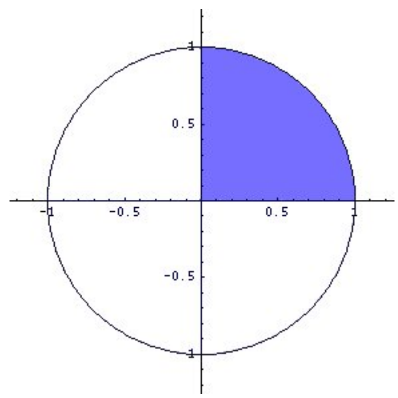
3. Consider the integral: $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3dydx$

(i) Compute the Integral.

This integral is very similar to the integrals we were computing in mathematica, only easier.

$$\int_0^1 \int_0^{\sqrt{1-x^2}} 3dxdy = 3\int_0^1 \sqrt{1-x^2}$$
$$= 3\int_0^1 \arcsin(x)dx$$
$$= \sqrt{1-x^2} + x\arcsin(x)\Big|_0^1 = \frac{3\pi}{4}$$

(ii) Sketch the region you are integrating over.



(iii) Change the order of integration.

This is something of a trick question. In this problem you can reverse the order of integration and have the integral

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3dxdy$$

(iv) Compute the new integral.

Since changing the order of integration did not change the integral there is no need to recompute the new integral.

4. Consider the integral: $\int_0^9 \int_{\sqrt{y}}^3 (x+y)dxdy$

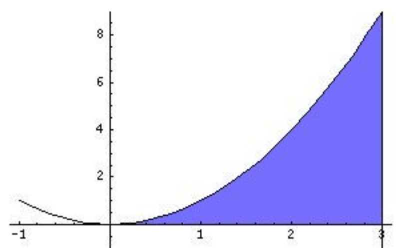
(i) Compute the Integral.

$$\int_0^9 \int_{\sqrt{y}}^3 (x+y)dxdy = \int_0^9 \frac{x^2}{2} + xy\bigg|_{\sqrt{y}}^3$$

$$= \int_0^9 \frac{9}{2} + \frac{5y}{2} - y^{3/2}dy$$

$$= \frac{1}{2}\left(9y + \frac{5y^2}{2} - \frac{4y^{5/2}}{5}\right)\bigg|_0^9 = \frac{891}{20}$$

(ii) Sketch the region you are integrating over.



(iii) Change the order of integration.

$$\int_0^3 \int_0^{x^2} (x+y)dydx$$

(iv) Compute the new integral.

$$\int_0^3 \int_0^{x^2} (x+y)dydx = \int_0^3 xy + \frac{x^2}{y}\bigg|_0^{x^2}$$

$$= \int_0^3 x^3 + \frac{x^4}{2}dx$$

$$= \frac{1}{20}\left(x^4(5+2x)\right)\bigg|_0^3 = \frac{891}{20}$$

And although it should not really be a surprise, the two integrals are equal.

5. Setup and evaluate the integral over the region V bounded by a cube, centered at $(0,1,2)$ on which all edges have length 2.

$$\iiint\limits_V xyz dV$$

The correct way to setup this integral is

$$\int_0^2 \int_1^3 \int_{-1}^1 xyz dx dz dy = \int_0^2 \int_1^3 \left. \frac{yzx^2}{2} \right|_{-1}^1 dz dy = \int_0^2 \int_1^3 0 dz dy = 0$$

So we find the value of the integral to be zero with no further work.

6. Evaluate the triple integral. $\int_{-1}^2 \int_1^{z^2} \int_0^{y+z} 3yz^2 dx dy dz$

$$\int_{-1}^2 \int_1^{z^2} \int_0^{y+z} 3yz^2 dz dy dz =$$

$$=3 \int_{-1}^2 \int_1^{z^2} \left. xyz^2 \right|_0^{y+z} dy dz =$$

$$=3 \int_{-1}^2 \int_1^{z^2} y^2z^2 + yz^3 dy dz =$$

$$=3 \int_{-1}^2 \left. \frac{y^3z^2}{3} + \frac{y^2z^3}{2} \right|_1^{z^2} dz =$$

$$=3 \int_{-1}^2 \frac{z^8}{3} + \frac{z^7}{2} - \frac{z^2}{3} - \frac{z^3}{2} dz =$$

$$=3 \left(\left. \frac{z^9}{27} + \frac{z^8}{16} - \frac{z^3}{9} - \frac{z^4}{8} \right|_{-1}^2 \right) = \frac{1539}{16}$$

So we have compute the value of this integral to be

$$\frac{1539}{16} \approx 96.1875$$