

1. Let  $C$  be the boundary of the surface  $z = x^2 + y^2$  with  $0 \leq x \leq 2$  and  $0 \leq y \leq 1$ , oriented with upward facing normal. Define  $\mathbf{F}(x, y, z) = (\sin(x^3) + xz, x - yz, \cos(z^4))$  and evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{s}$$

2. The helicoid surface  $S$  is parametrized by  $\mathbf{X}(s, t) = (s \cos(t), s \sin(t), t)$  for  $0 \leq s \leq 1$  and  $0 \leq t \leq \pi/2$ . Compute the line integral

$$\oint_{\partial S} \mathbf{F} ds$$

For the function  $\mathbf{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$

3. Let  $S$  be the hemisphere  $x^2 + y^2 + z^2 = 4$  with  $z \geq 0$  oriented upwards. Let  $\mathbf{F}(x, y, z) = (x^2 e^{yz}, y^2 e^{xz}, z^2 e^{xy})$  be a vector field. Evaluate:

$$\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$$

4. Let  $\mathbf{F}(x, y, z) = (xy, e^{z^2} + y, x + y)$  and let  $S$  be the graph of the function  $y = x^2/9 + z^2/9 - 1$  with  $y \leq 0$  oriented so that the normal vector  $S$  has positive  $y$  component. Compute the integral

$$\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$$

5. Use Stokes' Theorem to evaluate  $\oint \mathbf{F} \cdot ds$  where  $\mathbf{F}(x, y, z) = (y, z, x)$  and  $C$  is the triangle with vertices  $(0, 0, 0)$ ,  $(2, 0, 0)$  and  $(0, -2, 2)$  oriented counterclockwise when viewed from above.

6. The height of a mountain at a point  $(x, y)$  is given by  $z = 2 - x^2 - y^2$ . You walk counterclockwise around the mountain on the boundary of the region  $R$  defined by  $-1 \leq x \leq 1$  and  $-1 \leq y \leq 1$ . A magnetic field is exerting force  $\mathbf{F}(x, y, z) = (x^2y, 2x(y^2 + z), z^3)$  on your compass. Use Stokes' theorem to calculate the work done by the magnetic field as you walk around the mountain.