1. Find both first-order partial derivatives of both the functions

$$f(x,y) = e^{3y}\sin(x)$$
$$g(x,y) = xy^2\ln(3x)$$

2. Find the matrix of partial derivatives of the function

$$F(x,y,z) = (ze^{x^2+y^2} + xy, \cos(x^3y^2z^4))$$

3. A function $f(x,y)$ is harmonic if it satisfies the Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

   Show that $f(x,y) = x^3 - 3xy^2$ is harmonic.

4. The heat equation is: $\frac{\partial u}{\partial t} = k^2\frac{\partial^2 u}{\partial x^2}$. Show that $u(x,t) = e^{-k^2 t}\sin(x)$ is a solution of the heat equation.

5. Let $f(x, y) = x^2 + \frac{1}{2}y^2 - 2x$ Find a point on the graph $z = f(x, y)$ where the tangent plane is horizontal.

6. Let $f(x, y) = x/y + y/x$. Using a linear approximation about the point $(1/2, 1/4)$, estimate the value of $f(.48, .3)$.

7. An ant is trying to get out of the parabolic bowl $z = x^2 + 3y^2$. Suppose the ant is currently at the point $x = 2$, $y = -1$, $z = 7$. In which direction should the ant set out in order to climb out of the bowl fastest? Should it follow a straight line path from then on?